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MODERNIZATION OF THE METHODOLOGY AND INSTALLATION OF TRIAXIAL COMPRESSION FOR THE STUDY OF RHEOLOGICAL PROPERTIES OF DISPERSED MEDIA

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Abstract. The article discusses the determination of rheological and deformation characteristics of dispersed materials, specifically sand-clay molding mixtures. The authors present the design of a triaxial compression apparatus developed on the basis of a standard stabilometer, incorporating the principles of similarity theory. The study reveals that the presence of residual plastic deformations causes nonlinearity in the material layer formation process. Particular attention is paid to the analysis of the deformation modulus and its dependence on medium density, particle diameter and the internal friction coefficient. The results obtained allow for a more accurate prediction of the behavior of cohesive and non-cohesive bulk media under load, which is essential for optimizing technological processes in foundry production and related industries.

Keywords: dispersed materials, rheology, stabilometer, triaxial compression, deformation modulus, molding mixture.

1. Introduction

When processing dispersed and lump materials, both cohesive (such as molding mixtures) and non-cohesive (such as coke and limestone), their physical, mechanical, and rheological properties, including elasticity, plasticity, and strength, are crucial. One of the key parameters that determines the deformation behavior of dispersed media under load (including compression in the absence of lateral expansion) is the deformation modulus.

According to the literature data [1, 2], the value of the deformation modulus M is determined by the density of the material δ , the particle diameter s , the coefficient of internal friction γ , and a number of other factors, among which the three listed indicators have a dominant influence. It is known that an increase in the initial density of the medium naturally increases the deformation modulus. The coefficient of internal friction, which reflects the resistance of the medium to shear deformation during fracture, also increases as the packing of particles becomes denser, leading to an increase in the deformation modulus [3]. Conversely, the effect of dispersion (particle size) is the opposite: an increase in the average particle diameter causes a decrease in the deformation modulus.

Due to the multidirectional and interconnected nature of these factors, it is difficult to analytically predict their complex impact, which is critical for theoretical calculations and practical applications. Based on this, a plan was developed for experimental studies to quantify the dependence of the deformation modulus on key physical and mechanical characteristics of the medium.

2. Materials and Methods

To solve the research problems, a modernized experimental triaxial compression setup was assembled (Fig. 1). It allows for independent setting of the two principal stresses vertical τ_z and lateral $\tau_{r,y}$ and measurement of the corresponding principal relative strains j_z and $j_{r,y}$. Axial pressure on the specimen is exerted directly through a piston of the same diameter. As a result, when the specimen is compressed, its lateral expansion is possible only with the corresponding removal of some of the water filling the chamber. This device can be classified as a type stabilometer *B*, in which the lateral pressure is not simultaneously all-round, the axial pressure can be set independently of the lateral pressure and, in particular, be less than the latter or equal to zero. The stabilometer schematic is shown in Fig. 2. The instrument allows for compression testing, determining the lateral pressure coefficient and deformability characteristics, as well as the strength of the specimen [4].

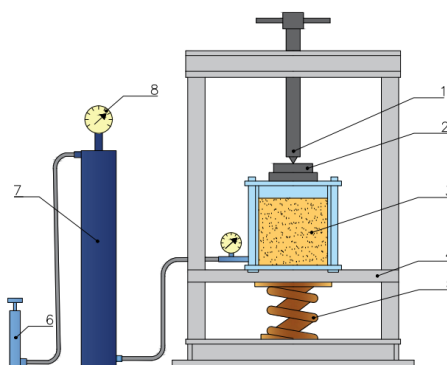


Fig. 1. The scheme of the experimental setup: 1-crane; 2-dynamometer; 3-stabilometer; 4-frame for attaching a type B stabilometer; 5-spring; 6-air pump for supplying air to the compensator tank; 7-compensator tank; 8-pressure gauge.

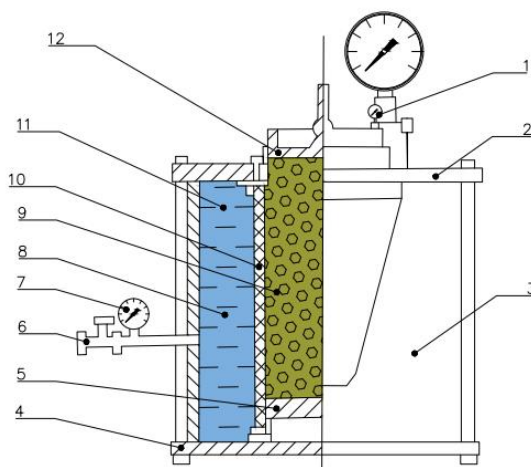


Fig. 2. Scheme of the experimental stabilometer: 1 – TdL 7682-1 dial gauge for measuring vertical deformations; 2 – upper cover; 3 – tap for regulating lateral pressure; 4 – lower cover; 5 – lower piston; 6 – tap for supplying water; 7 – pressure gauge for measuring lateral pressure; 8 – water inside the working chamber; 9 – sample; 10 – rubber cover; 11 – working chamber $H = 300mm, \varnothing = 400mm$; 12 – piston for vertical deepening of the sample $\varnothing = 136mm$.

2.1 Principle of Operation of the Experimental Setup

The design and experimental procedure for the developed testing apparatus (stabilometer) are as follows. The particulate material under study 9, pre-enclosed in an elastic rubber membrane 10 to prevent contact with the working medium, is placed inside the pressure chamber 11. The sealed specimen is rigidly fixed in the vertical direction between the lower 5 and upper 12 pistons (loading plungers). Following specimen installation, the internal volume of the chamber 11 is filled with distilled water. When the stopcock 3 is closed, the hydraulic system becomes completely sealed. Due to the practical incompressibility of water, the lateral (radial) expansion of the particulate material during vertical loading is restricted to zero, ensuring strict confined compression conditions. Consequently, the radial stress $\tau_r = \tau_y$ generated within the closed volume of the fluid due to material lateral pressure is continuously recorded by a standard pressure gauge 7. The measured hydraulic pressure is taken as the true lateral pressure of the medium under study.

2.2 Methodological Features of the Experimental Procedure

During the preparation and execution of the experiments, maintaining two key technological conditions is critical:

- Absolute tightness of the hydraulic circuit. Even minimal leakage of the working fluid from the chamber 11 leads to uncontrolled radial deformation of the specimen, which fundamentally distorts the measured values of the lateral pressure coefficient.
- Complete deaeration of the system. Air trapped within the water, the pressure gauge 7, and the connecting lines must be fully evacuated (via vacuuming or flushing with deaerated water). The presence of gas microbubbles, which possess high compressibility, violates the condition of rigid lateral confinement and significantly reduces the accuracy of deformation characteristics determination.

2.3 Analytical Capabilities of the Method

The developed design of the apparatus enables comprehensive compression testing of particulate media with stable, real-time parameter tracking. Based on the experimental results, the following parameters are determined:

- The coefficient of lateral earth pressure at rest (lateral pressure coefficient of the particulate material) ε ;
- Key deformation characteristics (modulus of deformation, coefficient of relative compressibility);
- The shear strength envelope (ultimate shear resistance) of the specimen under specified loading paths.

The limited height of the sample, in the presence of friction at the points of contact between the stamp and the bottom of the device, causes the sample to expand unevenly in height.

To eliminate these shortcomings, the authors [5] recommend that the height of the sample in the experiments be at least twice its diameter.

Taking into account the test methodology [6], compression tests were carried out according to two testing schemes, the difference between which was that at each stage of loading in the first scheme the vertical load was maintained constant, in the second – this load changed.

The experiments were conducted in the following order:

1. The stabilometer was loaded with material at a temperature corresponding to the ambient temperature in the laboratory.
2. The load was applied to the sample smoothly, avoiding impacts, in steps. The average rate of load increase was 30–50 kPa/min.
3. Each pressure step was maintained until the vertical deformation stabilized, not exceeding 0.0001 mm in 15 minutes.
4. Readings from the vertical and lateral pressure measuring devices, as well as the indicator, were recorded in the test log at each pressure stage 15 minutes after the set pressure was established.
5. Testing was terminated when the vertical pressure reached $P_b = 480$ kPa.
6. After completion of the test, the material sample was unloaded by resetting the vertical and lateral pressures to zero and removed from the stabilometer chamber.

The shear strength study on a triaxial apparatus was also conducted using two schemes. In the first, a fixed, constant lateral pressure was maintained throughout the experiment while the vertical load varied; in the second, after applying the vertical load and holding for ten minutes at the initial stage, the vertical load was increased until the specimen failed [7]. Subsequently, a high lateral pressure was created in the chamber, and

a gradually increasing vertical load was again applied to the specimen. Tests using both schemes were conducted at lateral pressures equal to 20, 40, 60, 80, and 100 kPa. It should be noted that the experiments using the second scheme were considered accelerated, since they were three times faster in duration compared to the tests using the first scheme.

As is known [8], for longitudinal deformations in the direction of the r , y , and z axes, in the general case of a three-dimensional stress-strain state, these dependencies are as follows:

$$j_r = \frac{\tau_r - u(\tau_y + \tau_z)}{M}, \quad j_y = \frac{\tau_y - u(\tau_r + \tau_z)}{M}, \quad j_z = \frac{\tau_z - u(\tau_r + \tau_y)}{M} \quad (1)$$

where parameters M and u are quantitative characteristics of the deformability of the medium under study.

A compression test involves the vertical deformation (compression) of a cylindrical sample of dispersed materials in a rigid ring (cage). The sample is in a uniform triaxial stress state characterized by normal stresses: $\tau_z = \tau$, $\tau_r = \tau_y = \varepsilon \tau_z$. Considering compression tests as a special case of triaxial deformation, when: $j_r = j_y = 0$, depending on (1) was found:

$$j_z = \frac{\tau}{M} \cdot \frac{1-2u^2}{1-u} \quad (2)$$

$$\varepsilon = \frac{u}{1-u} \quad (3)$$

It is easy to see that, based on the results of measurements of the compression stress Q and compression Δq of the sample, it is impossible to determine M and u separately; one can only find the value of the expression that contains both quantities simultaneously:

$$\frac{j_z}{\tau} = \frac{\Delta q}{q_0 \tau} = \frac{\frac{1-2u^2}{1-u}}{M} \quad (4)$$

where j_z – initial relative deformation of the sample before applying the current pressure stage (dimensionless quantity); τ – increment of vertical stress at this stage ($\tau = \tau_i - \tau_h$), kPa; τ_i – final (specified) vertical stress at the fixed loading stage, kPa; τ_h – initial vertical (axial) stress, kPa; Δq – absolute vertical strain (compression displacement) (mm); q_0 – initial height of the specimen (mm); $\frac{1-2u^2}{1-u}$ – lateral constraint coefficient (a coefficient accounting for the constraint of lateral expansion / rigid confinement in a stabilometer); u – Poisson's ratio; M – deformation modulus (elasticity/compaction modulus), kPa.

Usually, Poisson's ratio u is taken from reference data, and then M is found from relation (4). For the case of a limited pressure range $\tau = \tau_r - \tau_h$:

$$j_z = \frac{j_h - l_i}{1 + l_h} = \frac{b}{1 + j_h} \cdot (\tau_i - \tau_h) = \frac{b\tau}{1 + j_h} \quad (5)$$

where j_h – initial relative deformation (strain) of the sample before applying the current pressure stage; l_h – initial relative deformation of the sample before applying the current pressure step, (dimensionless quantity); l_i – final relative deformation of the sample at the controlled test stage, (dimensionless value); b – the compaction (compression) coefficient of a dispersed material, which characterizes the compliance of its framework and structural variability under external load, kPa⁻¹. From expressions (2) and (5) we obtain the relationship between M , u and b :

$$M = \frac{1 + j_h}{b} \cdot \left(\frac{1 - 2u^2}{1 - u} \right). \quad (6)$$

Statistical Analysis

All experiments were performed in at least three independent replicates ($n=3$). The experimental data were statistically analyzed using Microsoft Excel and Origin software. Results are presented as mean $\pm$ standard deviation ($\bar{x} \pm SD$) or mean with a 95% confidence interval. The variance (S^2) and standard error of the mean ($SE = \frac{SD}{\sqrt{n}}$) were calculated to verify data reproducibility. Final parameter estimation and functional approximations were performed using the multivariate least squares method.

3. Results

To validate the proposed methodology, a series of triaxial compression experiments was carried out on several dispersed materials. The obtained deformation and strength characteristics are presented below. The experimental results showed that the modulus M , as well as the lateral pressure and Poisson coefficients, largely depend on the porosity of the layer. These difficulties in determining M and u can be avoided by measuring the $\tau_r = \tau_y$ values at each loading stage. Then, from the $\tau_r = f(\tau_z)$ dependence graph (Fig. 3a) approximated by a straight line of the form $y = kx + b$, one can easily obtain the lateral pressure coefficient $\varepsilon = \frac{\tau_r}{\tau_z}$. Then, from the $j_z = f(\tau_z)$ dependence graphs, also approximated by a straight line (Fig. 3 b), one can determine the modulus $M = \frac{\Delta\tau_z}{\Delta j_z} \cdot \alpha$, $\alpha = \frac{1-2u^2}{1-u}$, where α is a coefficient that takes into account the impossibility of lateral expansion. Poisson's ratio can be determined as $u = \frac{\varepsilon}{1+\varepsilon}$. The values of these coefficients for some materials are given in Table1.

Table 1. MechMechanical properties of the investigated materials (mean $\pm$ SD, n = 3).

Parameter	Limestone Scheme 1	Limestone Scheme 2	Sand Scheme 1	Sand Scheme 2	Coke Scheme 1	Coke Scheme 2
Deformation modulus, M, MPa	1.58 $\pm$ 0.05	1.54 $\pm$ 0.04	1.95 $\pm$ 0.06	1.94 $\pm$ 0.05	7.67 $\pm$ 0.18	7.72 $\pm$ 0.21
Lateral pressure coefficient, dimensionless	0.75 $\pm$ 0.02	0.73 $\pm$ 0.02	0.39 $\pm$ 0.01	0.39 $\pm$ 0.01	0.25 $\pm$ 0.01	0.26 $\pm$ 0.01
Poisson's ratio, dimensionless	0.44 $\pm$ 0.01	0.42 $\pm$ 0.01	0.30 $\pm$ 0.01	0.30 $\pm$ 0.01	0.18 $\pm$ 0.01	0.21 $\pm$ 0.01

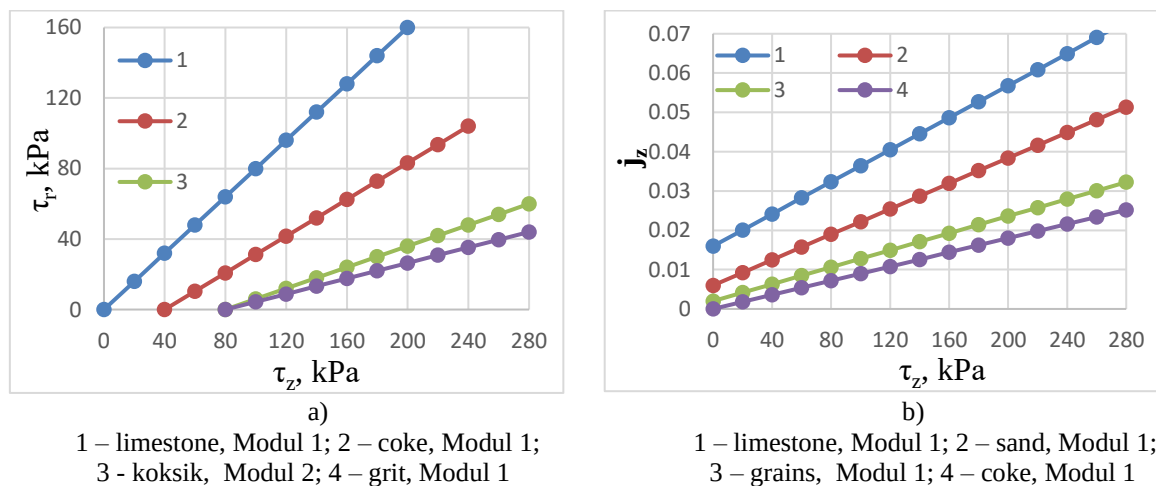


Fig. 3. The effect of vertical stress on the value of: a) lateral stress and b) relative deformation for different materials.

The primary characteristics of the reference materials used during the experimental validation are systematically presented in Table 2.

Table 2. The primary characteristics of the reference materials

Parameter	Sand	Iron ore concentrate	Coke	Limestone	Iron ore concentrate grit
Equivalent particle diameter, 10^{-3}m	0.16	0.08	2.00	0.63	2.00
Density, kg/m^3	2923	3547	1652	2801	3686
Hovering speed, m/s	2.8	1.6	7.9	5.8	9.2
Internal friction coefficient	1.42	0.92	0.94	0.55	0.51
Layer porosity, m^3/m^3	0.36	0.29	0.56	0.47	0.53

To determine the strength characteristics [9, 10], dependence graphs were constructed $j_z = f(\tau_z - \tau_r)$ at different values of lateral pressure τ_r (Fig. 4). The values were determined from the graphs $(\tau_z - \tau_r)$, corresponding to the moment of specimen destruction. Figure 4 shows the classic deformation curves of a

granular medium at four fixed levels of lateral pressure. The physical meaning of the curves is as follows: the graph clearly demonstrates the nonlinear nature of the medium's deformation. The point where the curve becomes almost vertical corresponds to the moment when the final equilibrium is reached and the sample breaks (shears). The greater the material's bearing capacity, the greater the stress deviation at which failure occurs $\tau_z - \tau_r$. These figures serve as an experimental confirmation that the presence of residual plastic deformations determines the strictly nonlinear nature of the formation and compaction of the layers of dispersed media. The coordinates of the fracture points (τ_z^p and τ_r^p) shown in this figure are directly used by the authors in the article for mathematical analysis.

The angle of internal friction and adhesion were determined as $\tan \gamma = \frac{F-1}{2\sqrt{F}}$, $c = \frac{T}{2\sqrt{F}}$, where F and T are coordinates determined from the expressions:

$$F = \frac{n \sum \tau_r^p \tau_z^p - \sum \tau_r^p \sum \tau_z^p}{n \sum (\tau_r^p)^2 - (\sum \tau_r^p)^2}; \quad T = \frac{1}{n} (\sum \tau_z^p - N \sum \tau_r^p) \quad (7)$$

where τ_z^p and τ_r^p are τ_z the τ_r values of and at specimen failure; n is the number of tests.

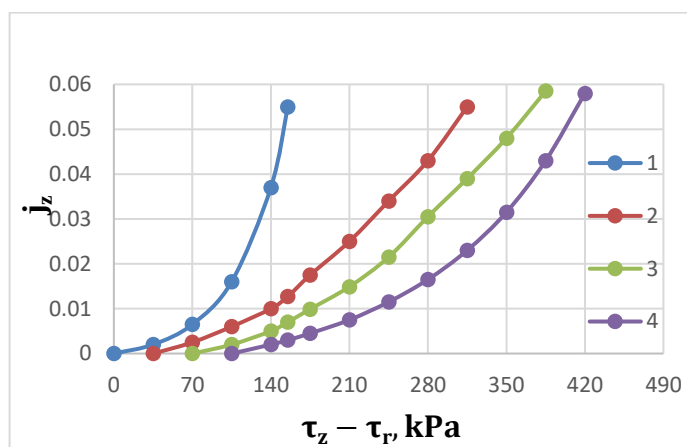


Fig. 4. Dependence of the relative strain of the grit (Modul 2) on the value of $\tau_z - \tau_r$:
1 – $\tau_r = 20$ kPa; 2 – 40 kPa; 3 – 60 kPa; 4 – 80 kPa.

The calculated parameter values are presented in Table 3. A comparison of experimental and known from the literature [11] values of the angle of internal friction ($\tan \gamma$) showed that their maximum difference does not exceed 10%, which indicates not only the qualitative but also the quantitative correspondence of the data we obtained to real conditions [12,13]. Based on this information, an experimental plan developed to evaluate the dependence of the deformation modulus on the above-mentioned parameters [14, 15].

Table 3. Strength characteristics of the investigated materials (mean $\pm$ SD, $n = 3$)

Parameter	Coke breeze Scheme 1	Coke breeze Scheme 2	Sand Scheme 1	Sand Scheme 2	Limestone Scheme 1	Limestone Scheme 2
Internal friction angle, φ , deg	43 $\pm$ 1.1	44 $\pm$ 1.2	31 $\pm$ 0.8	32 $\pm$ 0.9	35 $\pm$ 1.0	34 $\pm$ 0.9
Internal friction coefficient, $\tan \varphi$, dimensionless	0.92 $\pm$ 0.03	0.91 $\pm$ 0.03	0.61 $\pm$ 0.02	0.58 $\pm$ 0.02	0.70 $\pm$ 0.02	0.68 $\pm$ 0.02

The objective was to determine the influence of the main characteristics of the dispersed medium. In other words, the deformation modulus is a parameter. The factors chosen were material density δ , particle diameter s and the coefficient of internal friction γ . The number and values of the levels of the variable factors were determined (Table 4). The factor levels presented in Table 4 were established on the basis of the physical and mechanical properties of the investigated dispersed materials summarized in Table 2. The planning matrix was implemented by varying the density, equivalent particle diameter and internal friction angle within the experimentally observed ranges.

Table 4. Factor levels

Levels	Level 1	Level 2	Level 3	Level 4	Level 5
Particle diameter (s), mm	0.2	0.4	0.8	1.6	3.2
Density (δ), $\frac{kg}{m^3} \cdot 10^{-3}$	1.1	1.2	1.3	1.4	1.5
coefficient of internal friction (γ), deg	22	24	26	28	30

The choice of five levels motivated by the fact that the number of experiments is not very large (only 25) yet sufficient to identify the required pattern of dependence [16]. With three levels identifying the pattern of dependence is difficult due to the limited number of control points. Next, a planning matrix was compiled for the three-factor five-level experiment [17]. The planning matrix is shown in Table 5.

The following difficulty arose when planning the experiment. Since the coefficient of internal friction is related to density, not all levels could be filled with the same values. The experimental results are presented in Table 6. Experiments were conducted using a stabilometer. Each experiment was repeated at least three times, meaning it consisted of at least three trials. The arithmetic mean standard error and variance were determined using standard methods. In subsequent calculations the arithmetic mean values were used as the closest approximation to the actual values. Final data processing and approximation of the empirical relationships were carried out based on the multivariate least squares method [18].

Table 5. Planning matrix

Levels	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Density (δ), $\frac{kg}{m^3} \cdot 10^{-3}$	1	1	1	1	1	2	2	2	2	2	3	3	3	3	3	4	4	4	4	4	5	5	5	5	5
Particle diameter (s), mm	1	2	2	1	1	1	2	3	1	2	4	2	3	4	3	5	3	3	4	5	5	4	5	4	5
coefficient of internal friction (γ), deg	1	2	3	4	5	6	1	2	4	5	1	4	3	5	2	5	2	4	3	1	2	3	5	1	4

Table 6. Parameter values ($M \pm SD$, MPa)

Level	M_1	M_2	M_3	M_4	$\bar{M} \pm SD$
1	29.8	29.9	30.1	30	29.95±0.13
2	32.8	32.9	33.2	33	32.98±0.17
3	39.9	40.1	40.2	40	40.05±0.23
4	37.8	37.9	38.1	38	37.95±0.18
5	41.9	42.1	42.2	42	42.05±0.27
6	31.8	31.9	32.2	32	31.97±0.15
7	38.9	39.1	39.2	39	39.05±0.19
8	39.8	39.9	40.1	40	39.95±0.22
9	44.7	44.9	45.2	45	44.95±0.32
10	48.7	48.8	48.9	49	48.85±0.34
11	43.8	43.9	44.1	44	43.95±0.29
12	44.6	44.9	45.2	45	44.92±0.31
13	41.7	41.8	41.9	42	41.85±0.25
14	56.8	56.9	57.1	57	56.95±0.42
15	40.6	40.8	41.2	41	40.90±0.24
16	60.8	60.9	61.1	61	60.95±0.45
17	48.7	48.8	48.9	49	48.85±0.35
18	47.6	47.9	48.2	48	47.92±0.34
19	45.8	45.9	46.1	46	45.95±0.32
20	43.1	43.2	43.3	43	43.15±0.28
21	41.8	41.9	42.1	42	41.95±0.26
22	47.7	47.8	47.2	48	47.67±0.33
23	64.8	64.9	65.1	65	64.95±0.50
24	44.7	44.8	45.2	45	44.92±0.30
25	61.8	61.9	62.1	62	61.95±0.46

Note: Values are represented as mean $\pm$ standard deviation for $n \geq 3$ trials.

First paired dependencies were determined by sampling. After sampling the experimental results for each level of each factor five-point paired partial dependencies were obtained between the deformation modulus

and density the angle of internal friction and particle size [19]. The influence of factors on the deformation modulus is shown in Figure 5. Private point dependencies were obtained after ranking the results by increasing each factor and were described mathematically.

The corresponding correlation equations were found using the least squares method. They have the following form: $M_\gamma = 2\gamma - 9$, $M_\delta = 27.3 \cdot \delta^{1.8}$, $M_s = 40.1 \cdot s^{-0.2}$. These values were then substituted into the empirical multifactorial equation of Protodyakonov [20]:

$$y_n = \frac{\sum_{i=1}^k y_i}{y_{av}^{k-1}} \quad (8)$$

where y_n – multifactor equation of M.M. Protodyakonov; y_i – partial functions; y_{av} – average value of all experimental results taken into account; k – number of factors (partial functions).

Thus, we constructed a generalized formula that takes into account all factors:

$$M = \frac{M_\delta \cdot M_\gamma \cdot M_s}{(M_{av})^2} = \frac{27.3 \cdot \delta^{1.8} \cdot 40.1 \cdot s^{-0.2} \cdot (2\gamma - 9)}{2023.2004} = 0.54 \cdot s^{-0.2} \cdot \delta^{1.8} \cdot (2\gamma - 9) \quad (9)$$

where γ – angle of internal friction, *deg*; δ – material density, $\frac{\text{kg}}{\text{m}^3} \cdot 10^{-3}$; s – particle diameter, *mm*.

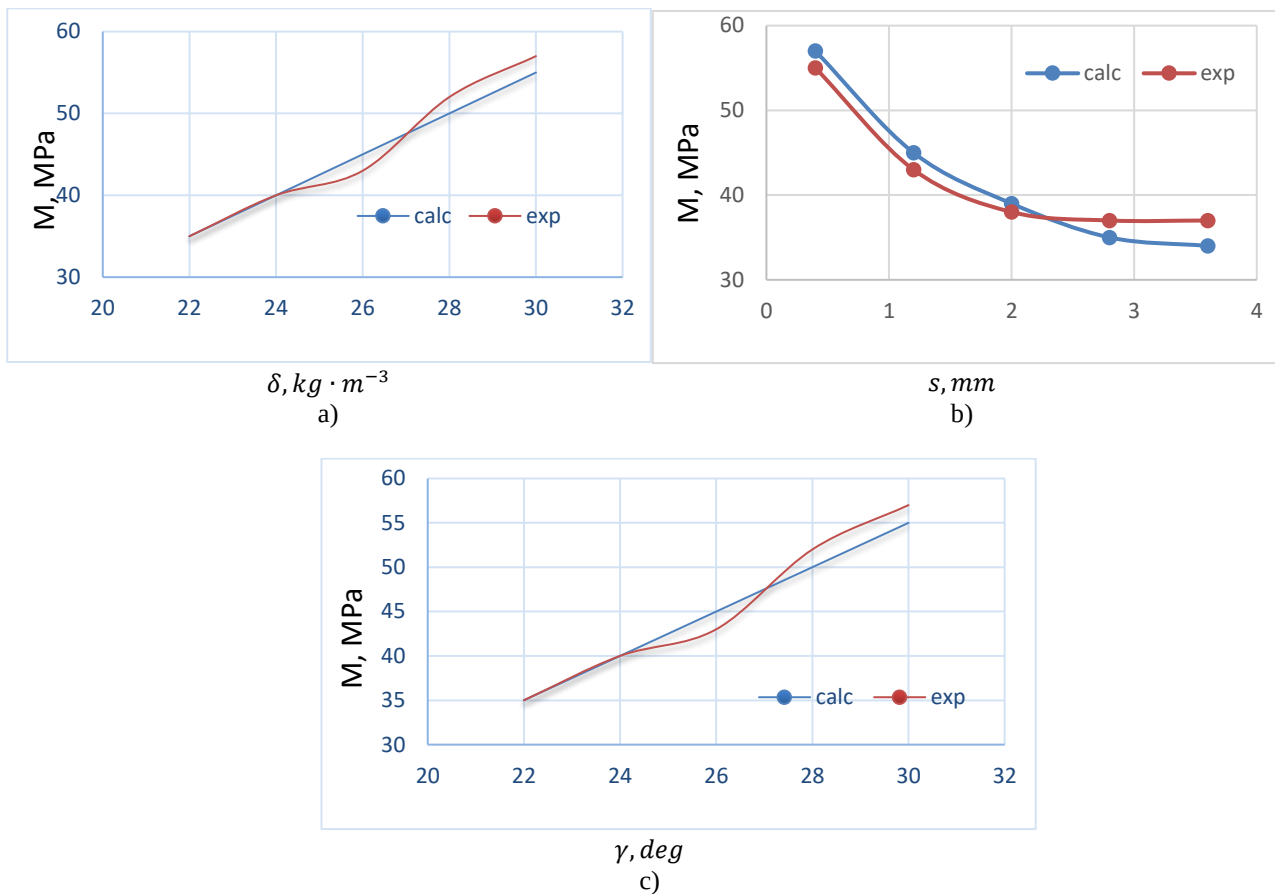


Fig. 5. Effect of deformation modulus:
a) density; b) particle size; c) the internal friction coefficient.

The minus sign indicates that the effect of the corresponding factor is inversely proportional: the greater the factor, the smaller the parameter M . An increase in the density of the granular medium leads to an increase in the deformation modulus. The increase in deformation with a decrease in particle size can be explained as follows: the smaller the material, the more densely it packs together, the greater the number of interparticle contacts, and the greater the force required for deformation.

To assess the goodness of fit, the nonlinear multiple correlation coefficient was used [21]:

$$R = \sqrt{1 - \frac{(n-1) \sum_{i=1}^n (y_{expi} - y_{thei})^2}{(n-k-1) \sum_{i=1}^n (y_{expi} - y_{avei})^2}} \quad (10)$$

where n – number of data points; k – number of active factors (equal to one for partial dependencies); y_{expi} – experimental value of the result; y_{thei} – theoretical (calculated) value; y_{avei} – the average experimental value.

Based on the calculation results, the correlation coefficient was found for $n=25$ and $k=3$. It is equal to $R=0.8$, and the significance is $t=10.17 > 2$, which indicates the adequacy of the generalized equation. The error of the equation was calculated using the formula and amounted to 11%:

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (y_{expi} - y_{thei})^2}{(n-k-1)}} \quad (11)$$

The experimental results showed that the deformation modulus is significantly influenced by material density, particle size, and the coefficient of internal friction. An increase in density and internal friction coefficient leads to higher stiffness of the medium, whereas an increase in particle diameter causes a decrease in the deformation modulus. The triaxial compression tests also demonstrated the nonlinear character of deformation and compaction processes caused by residual plastic strains. The determined strength and deformation characteristics are in good agreement with published data, with deviations not exceeding 10%, confirming the reliability of the proposed approach.

4. Discussion

The experimental results demonstrate that the deformation behavior of dispersed media under triaxial compression is governed primarily by three interrelated factors: material density, particle size, and the internal friction coefficient. An increase in density leads to a more compact particle arrangement and a larger number of interparticle contacts, resulting in higher resistance to deformation and, consequently, an increase in the deformation modulus. This tendency agrees with the classical concepts of granular media mechanics and soil behavior reported in previous studies [1–3].

The influence of particle size is opposite. As the equivalent particle diameter increases, the deformation modulus decreases because the contact area between particles is reduced and the packing becomes less dense. Similar relationships have been reported in studies devoted to the mechanical behavior of granular soils and particulate materials under compression [9–13]. The obtained results confirm that particle size should be considered together with density rather than as an independent parameter.

The internal friction coefficient also has a significant influence on the deformation characteristics. Materials possessing larger internal friction angles exhibit higher stiffness and greater resistance to irreversible deformation. This observation is consistent with the physical interpretation of shear resistance in granular systems and agrees with the published experimental data on triaxial compression and shear testing [5, 6, 11].

An important result of this study is the experimental confirmation of the nonlinear character of deformation during confined compression. The observed nonlinearity is associated with the accumulation of residual plastic strains and progressive rearrangement of particles inside the granular structure. As loading increases, the contact network evolves continuously, leading to changes in stress redistribution and stiffness. This behavior confirms that linear elastic models are insufficient for describing the deformation of dispersed media over a wide loading range. Unlike many previous investigations that primarily focused on determining mechanical characteristics of soils or granular materials, the present work emphasizes the modernization of the experimental methodology and testing apparatus.

The proposed triaxial compression setup provides independent control of axial and lateral stresses while ensuring continuous measurement of deformation parameters. Such an approach improves the reliability of determining deformation characteristics and extends the applicability of the stabilometer to a wider range of dispersed materials. In contrast to previous studies, the present work combines modernization of the experimental apparatus with a validated methodology for determining both deformation and strength characteristics of dispersed media within a single triaxial testing procedure. The adequacy of the obtained empirical model is supported by the multiple nonlinear correlation coefficient ($R=0.8$), indicating satisfactory

agreement between the calculated and experimental values. The error of the equation was calculated using the formula and amounted to 11% is acceptable for engineering applications involving heterogeneous granular media, where natural variability of material properties is unavoidable.

The proposed methodology may therefore be used not only for laboratory determination of deformation properties but also for engineering prediction of the mechanical behavior of cohesive and non-cohesive dispersed materials in metallurgical, mining and foundry technologies. Further studies should investigate the influence of moisture content, particle shape, loading rate and cyclic loading on the rheological characteristics of dispersed systems.

5 Conclusions

1. A modernized triaxial compression methodology based on a standard stabilometer has been developed for determining the rheological, deformation and strength characteristics of dispersed media under controlled loading conditions. The proposed experimental setup provides independent control of axial and lateral stresses, thereby improving the reliability and reproducibility of experimental measurements.

2. Experimental investigations demonstrated that the deformation modulus of dispersed materials is primarily governed by material density, equivalent particle diameter, and the internal friction coefficient. An increase in density and internal friction results in higher stiffness of the granular medium, whereas an increase in particle size leads to a reduction in the deformation modulus.

3. The triaxial compression tests confirmed the nonlinear character of deformation caused by progressive particle rearrangement and the accumulation of residual plastic strains. This behavior should be taken into account when describing the mechanical response of cohesive and non-cohesive dispersed materials under compression.

4. An empirical multifactor model describing the dependence of the deformation modulus on the governing parameters was developed using the results of a planned experiment. The adequacy of the model was confirmed by satisfactory agreement between the calculated and experimental data ($R = 0.8$), while the average prediction error did not exceed 11%.

5. The proposed methodology and the modernized experimental apparatus may be used for laboratory determination of deformation characteristics and for engineering prediction of the behavior of dispersed materials in metallurgical, foundry, mining and other technological processes involving bulk media. Further investigations should consider the influence of moisture content, particle shape and loading rate on the rheological behavior of dispersed systems.

Conflict of interest statement

The authors declare that they have no conflicts of interest in this work.

CRediT author statement

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Statement on the use of Artificial Intelligence.

The authors declare that no artificial intelligence tools were used to generate scientific content, results, or conclusions of this article.

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