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COSMOLOGICAL EVOLUTION DYNAMICS AND $Om(z)$ DIAGNOSTIC IN A SCALAR–FERMION MODEL WITH A GENERALIZED SCALE FACTOR

Ratbay A.*, Razina O., Tsyba P., Satbekova E.

L.N. Gumilyov Eurasian National University, Astana, Kazakhstan

*Corresponding author: aisara.ratbay@gmail.com

Abstract. A cosmological model of an expanding Universe with scalar–fermion interactions is considered. Our model is based on the interaction of scalar and fermion fields via a Yukawa-type coupling, which influences the development of cosmological expansion in its late stages. The generalized scale factor is used to study the evolution and dynamics of the system. This approach allows us to explore various expansion regimes and analyze the contribution of fields, namely, the scalar and fermion components, to the total energy density and pressure. The viability of the cosmological evolution model is studied using the dimensionless Hubble parameter and the $Om(z)$ diagnostic. The behavior obtained differs from the standard Λ CDM case, where the parameter $Om(z)$ remains constant. Our results show that the effective component of dark energy dynamically evolves as the Universe expands. Our analysis demonstrates that the interaction of scalar and fermion fields can significantly influence the history and evolution of the expansion and lead to stable cosmological behavior. The proposed model can be used for further studies of dark energy interaction scenarios.

Keywords: Scalar–fermion interaction, Yukawa coupling, cosmic acceleration, dark energy models, $Om(z)$ parameter.

1. Introduction

The physical nature of the accelerating expansion of the Universe remains one of the central open problems in modern cosmology [1–3]. Observational data indicate that the expansion rate increases in the late stages, while within the standard formulation of general relativity, such behavior cannot be explained without introducing an additional component. This motivates us to consider extended cosmological scenarios in which the effective energy content of the Universe changes and does not remain constant. In this context, scalar field models have proven particularly effective, as they offer a flexible framework capable of describing various evolutionary regimes of our Universe [4–6]. At the same time, we can usefully consider interactions between scalar and fermionic fields [7–13]. Coupling with fermionic matter makes additional contributions to the total energy density and pressure, thereby influencing the overall dynamics and evolution of the expansion [7,8]. Yukawa-type interactions represent one of the simplest and most consistent ways to account for such effects [12,13]. To analyze the expansion history in a model-independent manner, we typically use diagnostic quantities. One of the most informative tools is the parameter $Om(z)$, defined as [14,15]

$$Om(z) = \frac{E^2(z) - 1}{(1+z)^3 - 1}, \quad (1)$$

where the dimensionless Hubble function is given by

$$E(z) = \frac{H(z)}{H_0}. \quad (2)$$

This diagnostic parameter allows us to track the evolution of the expansion rate and distinguish between different dark energy scenarios without specifying a specific cosmological model. A cosmological model with scalar-fermion interactions is considered within a framework that uses a generalized form of the chosen scale factor. Our goal is to investigate how the interaction term influences the expansion dynamics so that we can analyze the resulting behavior using the diagnostic parameter $Om(z)$.

2.Theoretical Framework and Research Methods

2.1. Field equations and Cosmological Dynamics

A cosmological model in which a scalar field is coupled to a fermionic sector is considered. Here we will work only with the scalar-fermionic system and neglect any additional contribution of matter. Our system dynamics are described by the action [7,8,12,13]

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - U(\phi) - \lambda \bar{\psi} \phi \psi + \frac{i}{2} (\bar{\psi} \Gamma^\mu D_\mu \psi - (D_\mu \bar{\psi}) \Gamma^\mu \psi) - V(\bar{\psi} \psi) \right] \quad (3)$$

where, ϕ is the scalar field, ψ is the fermionic spinor field, and $\bar{\psi}$ is its Dirac conjugate.

The function $U(\phi)$ denotes the scalar field potential, while $V(\bar{\psi} \psi)$ describes the fermionic self-interaction potential. The parameter λ characterizes the interaction between the scalar and fermionic sectors. To do this, we use the Friedman-Lemaître-Robertson-Walker spatially flat metric [4]

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2), \quad (4)$$

where, $a(t)$ is the cosmological scale factor describing the expansion dynamics of the Universe.

The effective Lagrangian takes the form

$$L = -3a\dot{a}^2 + a^3 \left[\frac{1}{2} \dot{\phi}^2 - U(\phi) - \lambda \bar{\psi} \phi \psi + \frac{i}{2} (\bar{\psi} \gamma^0 \dot{\psi} - \dot{\bar{\psi}} \gamma^0 \psi) - V(\bar{\psi} \psi) \right], \quad (5)$$

here, the dot denotes differentiation with respect to the cosmic time t . Together with the Friedmann–Robertson–Walker metric (4), the equations of motion derived from the action (3) take the following form

$$3H^2 = \rho, \quad (6)$$

$$-2\dot{H} - 3H^2 = p, \quad (7)$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dU}{d\phi} + \lambda \bar{\psi} \psi = 0, \quad (8)$$

$$\dot{\psi} + \frac{3}{2} H \psi + i \gamma^0 \frac{\partial V}{\partial \bar{\psi}} + i \lambda \gamma^0 \psi \phi = 0, \quad (9)$$

$$\dot{\bar{\psi}} + \frac{3}{2} H \bar{\psi} - i \frac{\partial V}{\partial \psi} \gamma^0 - i \lambda \phi \bar{\psi} \gamma^0 = 0 \quad (10)$$

The effective energy density and pressure are given by

$$\rho = \frac{1}{2} \dot{\phi}^2 + U(\phi) + \lambda \phi \psi \bar{\psi} + V(\bar{\psi} \psi), \quad (11)$$

$$p = \frac{1}{2} \dot{\phi}^2 - U(\phi) + \bar{\psi} \psi \frac{dV}{d(\bar{\psi} \psi)} - V(\bar{\psi} \psi). \quad (12)$$

First, the Dirac equations are solved, and the evolution equation for the fermionic bilinear invariant is then obtained:

$$\frac{d}{dt}(\bar{\psi}\psi) + 3H(\bar{\psi}\psi) + i\left(\bar{\psi}\gamma^0\frac{\partial V}{\partial\bar{\psi}} - \frac{\partial V}{\partial\psi}\gamma^0\psi\right) = 0. \quad (13)$$

For the nonlinear fermionic self-interaction [7,8]

$$V = \delta_0(\bar{\psi}\psi)^n, \quad (14)$$

here, δ_0 is a constant parameter and n determines the power-law form of the fermionic self-interaction potential. This equation gives

$$\bar{\psi}\psi = \frac{C}{a^3}, \quad (15)$$

where C is an integration constant. Therefore, the fermionic potential becomes to

$$V = \delta_0\left(\frac{C}{a^3}\right)^n = \frac{\delta_0 C^n}{a^{3n}}. \quad (16)$$

Substituting Friedmann equations, we get

$$3H^2 = \frac{1}{2}\dot{\phi}^2 + U(\phi) + \frac{c\lambda\phi}{a^3} + \frac{\delta_0 C^n}{a^{3n}}, \quad (17)$$

$$-2\dot{H} - 3H^2 = \frac{1}{2}\dot{\phi}^2 - U(\phi) + (n-1)\frac{\delta_0 C^n}{a^{3n}}. \quad (18)$$

2.2. Reconstruction of the Scalar-Fermion System

The generalized scale factor [9] is used in the analysis.

$$a(t) = \alpha + \beta e^{\gamma t^2} t^{2\delta}, \quad (19)$$

where the constants α, β, γ and δ control the dynamical behavior of the expansion.

This combination of the exponential and power-law scale factor goes beyond simple power-law models and allows for a richer set of evolutionary modes. At the same time, the exponential has the property of a super-de Sitter solution due to t^2 , characterized by faster dynamics. Figure 1 illustrates the dynamical evolution of the generalized scale factor with cosmic time.

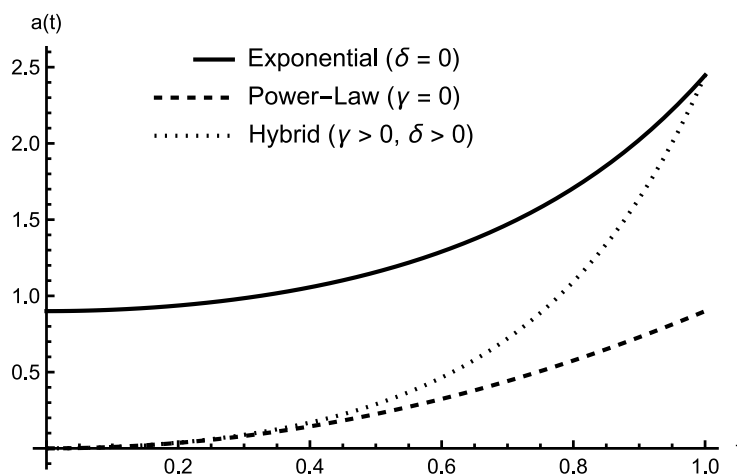


Fig.1. Evolution of the hybrid scale factor $a(t)$ for different parameter values: the exponential expansion regime ($\delta = 0$), the power-law expansion regime ($\gamma = 0$, dashed curve), and the hybrid expansion regime ($\gamma > 0, \delta > 0$, dotted curve).

For all considered parameter sets, the scale factor increases monotonically, describing an expanding Universe. Larger values of the parameter β lead to a more rapid growth of $a(t)$, indicating an enhanced expansion rate. At later stages, the expansion becomes significantly accelerated due to the exponential contribution in the chosen parametrization.

The corresponding Hubble parameter is

$$H = \frac{2\beta e^{\gamma t^2} t^{2\delta-1} (\gamma t^2 + \delta)}{\alpha + \beta e^{\gamma t^2} t^{2\delta}} \quad (20)$$

The derivative of the Hubble parameter follows directly from the explicit form of $H(t)$:

$$\dot{H} = \frac{\beta e^{\gamma t^2} t^{2\delta-2} [4\gamma^2 t^4 + 2\gamma(4\delta+1)t^2 + 2\delta(2\delta-1)] (\alpha + \beta e^{\gamma t^2} t^{2\delta}) - 4\beta^2 e^{2\gamma t^2} t^{4\delta-2} (\delta + \gamma t^2)^2}{(\alpha + \beta e^{\gamma t^2} t^{2\delta})^2}. \quad (21)$$

To obtain an explicit expression for the scalar field, we define the form of self-interaction potential as

$$U(\phi) = \frac{1}{2} m^2 \phi^2. \quad (22)$$

For convenience, we combine the Friedmann equations by subtracting the pressure equation from the energy density equation, which leads to

$$\rho - p = 6H^2 + 2\dot{H}. \quad (23)$$

Using the previously obtained expressions for ρ and p , we rewrite the left-hand side as

$$\rho - p = m^2 \phi^2 + \frac{c\lambda}{a^3} \phi + (2-n) \frac{\delta_0 c^n}{a^{3n}}. \quad (24)$$

By equating both expressions, we obtain the following algebraic equation for the scalar field

$$m^2 \phi^2 + \frac{c\lambda}{a^3} \phi + (2-n) \frac{\delta_0 c^n}{a^{3n}} - (6H^2 + 2\dot{H}) = 0. \quad (25)$$

The corresponding solution for the scalar field is [13]

$$\phi = \frac{-\frac{c\lambda}{a^3} \pm \sqrt{\frac{c^2 \lambda^2}{a^6} - 4m^2 \left[(2-n) \frac{\delta_0 c^n}{a^{3n}} - (6H^2 + 2\dot{H}) \right]}}{2m^2}. \quad (26)$$

Substituting the obtained the scalar field solution into the potential yields

$$U(\phi) = \frac{1}{8m^2} \left(-\frac{c\lambda}{a^3} \pm \sqrt{\frac{c^2 \lambda^2}{a^6} - 4m^2 \left[(2-n) \frac{\delta_0 c^n}{a^{3n}} - (6H^2 + 2\dot{H}) \right]} \right)^2. \quad (27)$$

3. Results and Discussion

3.1. Observational constraints

To work with observational data, a transition to redshift terms is required. The cosmological time t is not tied to a specific scale, which makes it difficult to determine the exact time intervals. The redshift z , on the contrary, allows for an accurate determination of the moment of an event. The transition to redshift terms is made according to the formula

$$a(t) = \frac{1}{1+z}. \quad (28)$$

The transition to redshift terms leads to a dimensionless Hubble parameter of the form

$$E(z) = \frac{2\beta\sqrt{\gamma}\delta e^{\delta\mathcal{W}}(1+\mathcal{W})}{\left(\beta e^{\delta\mathcal{W}} + \alpha\left(\sqrt{\frac{\delta}{\gamma}}\mathcal{W}\right)^{-2\delta}\right)\sqrt{\mathcal{W}}}, \quad (29)$$

where $\mathcal{W} = W\left(\frac{\gamma}{\delta}\left(\frac{(1+z)^{-1}-\alpha}{\beta}\right)^{\frac{1}{\delta}}\right)$ is the Lambert function.

However, working with observational data requires dimensional calibration of the Hubble parameter, so $E(z)$ is transformed using the formula $H(z) = H_0 E(z)$, where H_0 is the value of the Hubble parameter in the modern era at $z = 0$. The following sets of observational data are used to verify the model and select parameters:

1. *Cosmic Chronometers* (CC) [16] is a set of model-independent measurements of the Hubble $H(z)$ function obtained from the differential ages of passive galaxies in the range up to about $z \sim 2$.

2. *Baryonic Acoustic Oscillations* (BAO) datasets SDSS and DESI [17] contain measurements of the characteristic scale of baryon acoustic oscillations in the distribution of galaxies and quasars, usually in the form of combinations of distances (angular and radial) and the Hubble scale at different redshifts. They provide geometric constraints on cosmological parameters through the standard BAO ruler.

3. *Pantheon+ with SH0ES* [18] is a combined data set of type Ia Pantheon+ supernovae with local calibration of distances from SH0ES, providing measurements of the distance module in a wide range of redshifts and direct reference to the Hubble H_0 scale. The likelihood formula is used for calculations:

$$\ln \mathcal{L} = -\frac{1}{2}\chi^2 = -\frac{1}{2}\sum_i \left(\frac{H_{\text{mod}}(z_i) - H_{\text{obs}}}{\sigma_{H_{\text{obs}}}}\right)^2, \quad (30)$$

where H_{mod} is the Hubble function of the model, H_{obs} is the observational Hubble parameter and $\sigma_{H_{\text{obs}}}$ is the error of observations.

The Cobaya MCMC Bayesian analysis code with the Gelman-Rubin convergence criterion $R - 1 < 0.01$ was used to analyze the parameter values. The calculation results are shown in Figure 2 and Table 1.

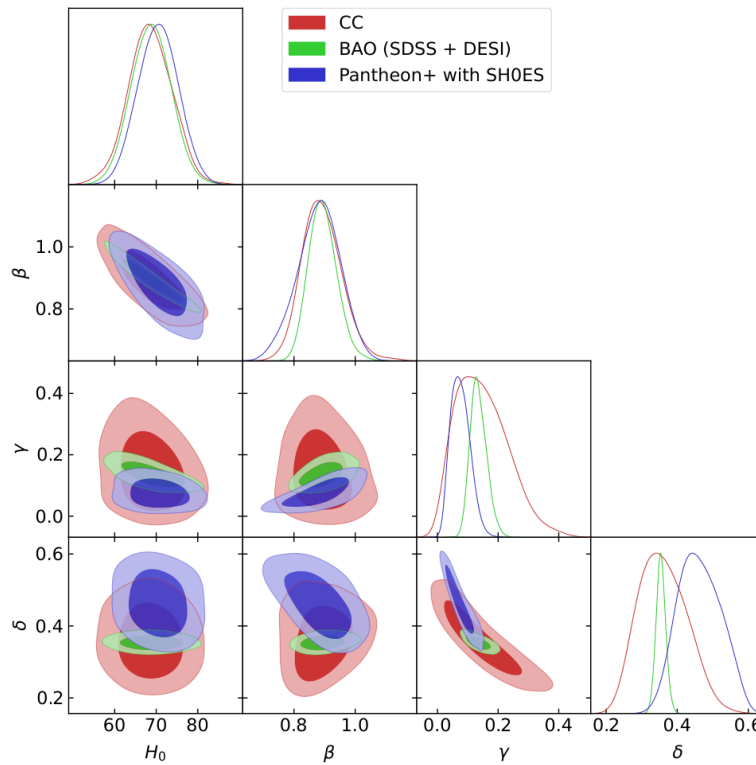


Fig.2. Constraints on cosmological parameters obtained from three independent datasets: CC (red), BAO (green) and Pantheon+ with SH0ES calibration (blue) with confidence levels 1σ and 2σ .

Table 1. Parameter values in the 1σ confidence interval

Dataset	$H_0, \text{m s}^{-1} \text{Mpc}^{-1}$	β	γ	δ
CC	$68.608^{+5.435}_{-5.241}$	$0.892^{+0.064}_{-0.064}$	$0.146^{+0.089}_{-0.090}$	$0.359^{+0.072}_{-0.070}$
BAO (SDSS+DESI)	$68.793^{+4.743}_{-4.885}$	$0.894^{+0.046}_{-0.046}$	$0.135^{+0.028}_{-0.027}$	$0.354^{+0.014}_{-0.013}$
Pantheon+ with SH0ES	$70.472^{+4.836}_{-4.929}$	$0.882^{+0.071}_{-0.070}$	$0.075^{+0.032}_{-0.033}$	$0.463^{+0.066}_{-0.061}$

3.2. Scalar-Field Evolution

Since the values of the scale factor parameters are known, the next step is to study the dynamics of the scalar field [9–13], the results of which are shown in Figure 3.

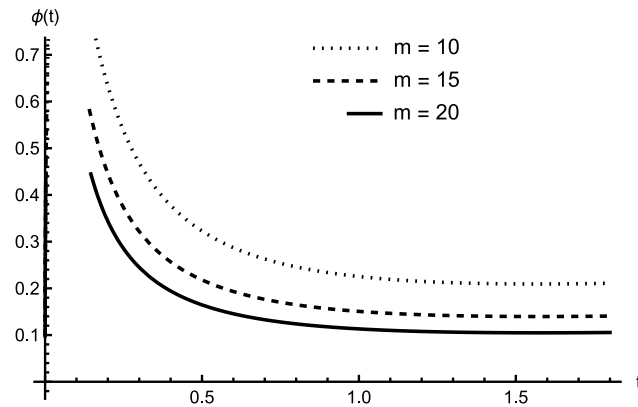


Fig.3. Evolution of the scalar field for different values of the parameter m : $m = 10$ (dotted), $m = 15$ (dashed), and $m = 20$. The scale-factor parameters are determined from the mean values reported in Table 1. The remaining model parameters are fixed at $C = 0.5$, $\lambda = 0.8$, $n = 2$, and $\delta_0 = 0.3$.

Figure 3 shows the time evolution of the scalar field. The field changes more rapidly at early stages, while at later times its variation becomes smoother, indicating a transition toward a more stable dynamical regime.

3.3. Energy Density and Pressure Evolution

The total energy density is given by the Friedmann equation

$$\rho = 3H^2. \quad (31)$$

Using Eq. (20), the total energy density becomes

$$\rho = 3 \left[\frac{2\beta e^{\gamma t^2} t^{2\delta-1} (\delta + \gamma t^2)}{\alpha + \beta e^{\gamma t^2} t^{2\delta}} \right]^2. \quad (32)$$

The bosonic energy density is

$$\rho_b = \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} m^2 \phi^2, \quad (33)$$

where the scalar field $\phi(t)$ is given by (26).

The Yukawa interaction contribution to the energy density is

$$\rho_{Yukawa} = \frac{c\lambda}{a^3} \phi. \quad (34)$$

Using Eqs. (19) and (26), the Yukawa contribution becomes

$$\rho_{Yukawa} = \frac{-\frac{C^2\lambda^2}{a^6} \pm \frac{C\lambda}{a^3} \sqrt{\frac{C^2\lambda^2}{a^6} - 4m^2 \left[(2-n) \frac{\delta_0 C^n}{a^{3n}} - (6H^2 + 2\dot{H}) \right]}}{2m^2} \quad (35)$$

This expression is obtained by substituting the scalar-field solution into the Yukawa interaction term, which explicitly reveals how the interaction contribution is governed by the scalar-field dynamics and its coupling to the fermionic sector [12,13]. The fermionic energy density is

$$\rho_{fermion} = \frac{\delta_0 C^n}{a^{3n}}. \quad (36)$$

Using Eq. (19), we obtain

$$\rho_{fermion} = \frac{\delta_0 C^n}{(\alpha + \beta e^{\gamma t^2} t^{2\delta})^{3n}}. \quad (37)$$

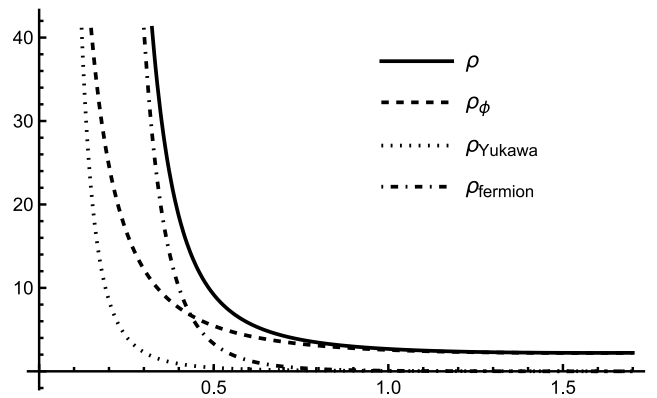


Fig.4 The time evolution of the energy density $\rho(t)$ and its components: ρ_ϕ (dashed), ρ_{Yukawa} (dotted) and $\rho_{fermion}$ (dot-dashed) with constant values: $C = 0.5, \lambda = 0.8, n = 2, m = 20$, and $\delta_0 = 0.3$.

Figure 4 shows how the total energy density and its components evolve with cosmic time. All curves decrease with increasing t , indicating a decrease in the effective energy density due to cosmic expansion and the gradual suppression of interaction terms in the late stages of the Universe. The Yukawa and fermion terms decrease rapidly and become almost negligible at later stages.

In contrast, the boson part changes more slowly and largely determine the characteristic behavior of the total energy density. This indicates that the contribution of the scalar field dominates the dynamics at later stages, while the other terms primarily influence the early stages of evolution rather than the later ones [4–6]. The pressure of the scalar field is

$$p_\phi = \frac{1}{2} \dot{\phi}^2 - \frac{1}{2} m^2 \phi^2. \quad (38)$$

where $\phi(t)$ is defined by Eq. (26).

The fermionic pressure is

$$p_{fermion} = (n-1) \frac{\delta_0 C^n}{a^{3n}}. \quad (39)$$

Using Eq. (19), the following expression is obtained:

$$p_{fermion} = (n-1) \frac{\delta_0 C^n}{(\alpha + \beta e^{\gamma t^2} t^{2\delta})^{3n}}. \quad (40)$$

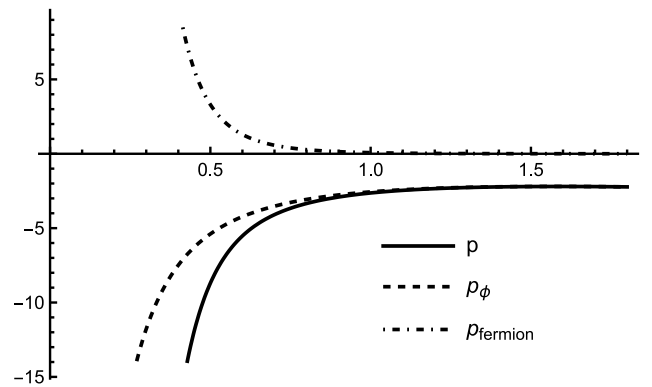


Fig.5. The time evolution of the energy density $p(t)$ and its components: p_ϕ (dashed), and $p_{fermion}$ (dot-dashed) with constant values $C = 0.5, \lambda = 0.8, n = 2, m = 20$, and $\delta_0 = 0.3$.

Figure 5 shows how the pressure component behaves in the model under consideration. In the early stages, we can see that the total pressure takes negative values and then gradually approaches zero as the system evolves. The scalar sector determines the overall dynamics, while the fermion contribution is secondary and rapidly diminishes. This behavior reflects the diminishing role of material structures during the cosmic expansion of the Universe.

3.4. $Om(z)$ Diagnostic Analysis

In this section, we analyze the evolution of the proposed cosmological model using the diagnostic function $Om(z)$, which provides a convenient tool for examining the expansion dynamics and distinguishing between different dark energy scenarios without specifying a particular model [14,15]. Using the Hubble parameter given in Eq. (20), the dimensionless Hubble function can be written as (29). The obtained expressions make it possible to study the evolution of $E(z)$ and compare it with the standard Λ CDM cosmology. The Om diagnostic parameter is defined as (1)

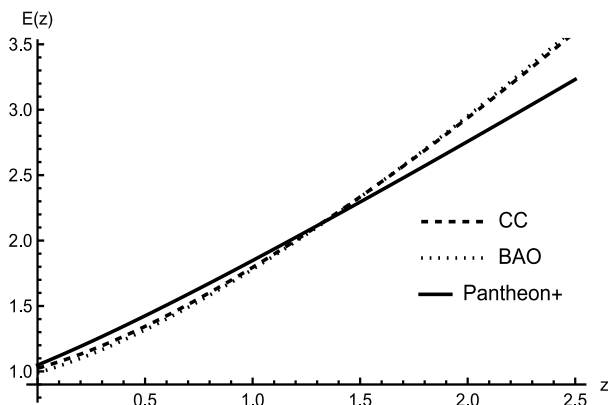


Fig.6. The function $E(z)$ as a function of the redshift z in the interval $0 < z \leq 2.5$

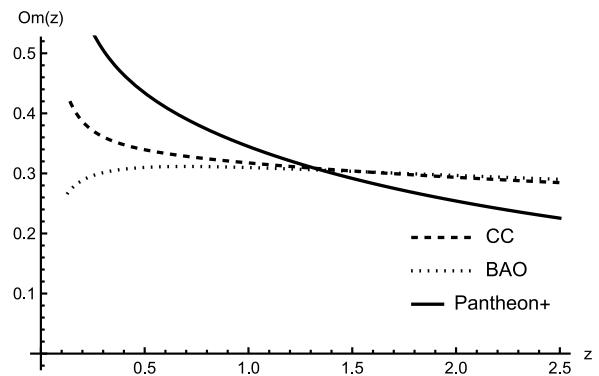


Fig.7. The function $Om(z)$ as a function of the redshift z in the interval $0 < z \leq 2.5$

It is observed that the curves exhibit a pronounced variation at small values of z , indicating an active dynamical regime at late times. As z increases, the behavior gradually transitions toward a nearly constant and stable profile. Such a trend suggests that the considered model deviates from the standard Λ CDM scenario, for which the $Om(z)$ parameter remains constant over redshift. The behavior of the parameter $Om(z)$ gives us important insight into the phenomenon of dark energy within the framework of our proposed model [14]. We observed deviations from constant behavior, indicating a deviation from the constant Λ CDM scenario, suggesting the presence of a dynamic component to dark energy [14,15].

4. Conclusions

Using a generalized scale factor, we analyzed a cosmological model with scalar-fermion interactions. This approach allowed us to describe a broader class of expansion regimes without imposing any restrictive assumptions on the dynamics of our Universe.

The field equations of the model are obtained, and the corresponding expressions for the energy density and pressure of all relevant components are derived. Our results show that the interaction between the scalar and fermion sectors leads to a nontrivial modification of cosmological evolution, especially in the late stages of the Universe.

Our reconstructed Hubble function $E(z)$ exhibits smooth and physically consistent behavior over the redshift range under consideration. However, we discovered that the diagnostic parameter $Om(z)$ does not remain constant, indicating a clear deviation from the standard Λ CDM scenario. The results obtained indicate the dynamical nature of the effective dark energy component in the model. The behavior we obtained suggests that the scalar field makes the dominant contribution to the overall dynamics in the late stages of evolution, while the fermionic component becomes secondary as the Universe expands.

The proposed model provides a consistent phenomenological description of cosmic evolution and can serve as an alternative approach to modeling dark energy using interacting fields [9–13].

Conflict of interest statement.

The authors declare that there are no conflicts of interest regarding the publication of this paper.

CRediT author statement.

Ratbay A.: Conceptualization, Investigation, Formal analysis, Writing – original draft; **Razina O.:** Methodology, Supervision, Validation, Writing – review & editing; **Tsyba P.:** Theoretical framework development, Formal analysis, Interpretation of results; **Satbekova E.:** Literature review, Data curation, Visualization. The final manuscript was read and approved by all authors.

Statement on the use of Artificial Intelligence.

Artificial intelligence tools were used only for language editing and formatting assistance. All scientific derivations, calculations, interpretations, and conclusions were obtained and verified by the authors.

Data Availability Statement

The data that support the findings of this article are openly available.

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AUTHORS' INFORMATION

Ratbay, Aisara — PhD student, Department of General and Theoretical Physics, L.N. Gumilyov Eurasian National University, Astana, Kazakhstan; <https://orcid.org/0009-0008-1031-9177>; aisara.ratbay@gmail.com

Razina, Olga — PhD, Professor, Department of General and Theoretical Physics, L.N. Gumilyov Eurasian National University, Astana, Kazakhstan; Scopus ID: 52364540700; <https://orcid.org/0000-0002-4400-4789>; olvikraz@mail.ru

Tsyba, Pyotr — PhD, Associate Professor, Department of General and Theoretical Physics, L.N. Gumilyov Eurasian National University, Astana, Kazakhstan; Scopus ID: 36946423600; <https://orcid.org/0000-0003-4928-0392>; pyotrtsyba@gmail.com

Satbekova, Enilik — Master's student, Department of General and Theoretical Physics, L.N. Gumilyov Eurasian National University, Astana, Kazakhstan; <https://orcid.org/0009-0007-6543-2105>; satbekova.enilik@bk.ru