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Research Article

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A DYNAMICAL SYSTEM APPROACH TO LANGUAGE BIAS EVOLUTION ON COMPLEX NETWORKS

Baibolatov Y.Z., Nalibayev Y.D., Kozhagulov Y.T.*

Al-Farabi Kazakh National University, Almaty, Kazakhstan
**Corresponding author:* eldos.kozhagulov@kaznu.kz

Abstract. *We propose a dynamical systems model to study language competition and bias evolution in structured agent populations. Each agent is characterized by a continuous bias variable representing their linguistic preference, evolving under the combined influence of peer interactions, native language retention, and external prestige forces. The model incorporates a nonlinear damping mechanism that confines the agent's bias within a fixed range between negative one and one, and allows for heterogeneous susceptibility and retention parameters. We analyze the model in its linear regime and perform a stability analysis of the fixed points under both symmetric and asymmetric network topologies. Simulations on fully connected and small-world networks reveal diverse dynamical scenarios, including language death, bilingual persistence, and spontaneous population bifurcation into opposing linguistic groups. The results provide insight into the interplay of social structure, identity, and external influence in shaping language dynamics.*

Keywords: Nonlinear dynamics, complex systems, networks.

1. Introduction

The evolution of languages in a globalized, interconnected society is shaped by complex interactions between individuals, their cultural identities, and external sociopolitical forces. As some languages grow in dominance while others face extinction, mathematical modeling offers a powerful framework to understand the mechanisms driving these dynamics. Previous studies have highlighted mechanisms of language death [1], bilingual coexistence [2], and the effects of social structure [3–10]. Language dynamics have also been explored using agent-based models [11], adaptive networks [12], and hybrid learning schemes [5,13]. The inclusion of prestige effects [1,14], inter-linguistic similarity [2], and stochasticity [13,15] enriches the modeling landscape. The study of language dynamics offers a compelling application of nonlinear dynamical systems, a core area in technical and applied physics. Our approach formalizes language bias evolution using continuous variables and differential equations on complex networks, employing techniques common in statistical physics, control theory, and systems engineering. Furthermore, the model's structure—governed by agent-level dynamics and influenced by network topology—parallels the analysis of synchronization [16], signal propagation, and collective behavior in engineered and physical systems [17–21]. This cross-disciplinary perspective aligns with the broader goal of applying physical modeling paradigms to complex social and technical systems.

In this work, we present a novel model of language bias evolution in agent-based populations, incorporating network-based peer influence, native language retention, and external prestige effects. The goal is to identify conditions that lead to outcomes such as language death, bilingualism, or stable language coexistence.

2. The Model

We consider a system of agents that speak either language A or language B, or both. Since in reality, even if people are bilingual, it is quite often that their language competence is not absolute for both languages. For example, if a person's native language is A, and at some point in his life, the person learns the language B, the level of competence is rarely the same as a native level. In this work, we construct a mathematical model of agents, with a dynamic variable being the "bias" ϕ_k towards one or the other language: $\phi_k = +1$ if the person speaks only language A, $\phi_k = -1$ if the person speaks only language B, and $-1 \prec \phi_k \prec +1$ if the person is bilingual. If the person perfectly speaks both languages at the same level, the value of $\phi_k = 0$, which means that the person does not have any preference in choosing the language. If the value of ϕ_k is positive, then we say that the person is bilingual, but with a preference for language A.

How does this bias change over time? The first and foremost purpose of a human language is communication with other people, and the bias naturally changes due to the social connections of the person. If a group of people speaks the same language at the same level, their bias and proficiency level do not change, hence $\dot{\phi}_k = 0$ for the agents of this group (a dot over a variable denotes derivative over time). But if a group of people with different levels of language proficiency and bias are connected, for example, a foreigner with an intermediate level of language proficiency is connected to a group of native speakers, then there is a natural drive to change the bias $\dot{\phi}_k \neq 0$. To model this behavior, we consider a simple diffusive coupling model

$$\dot{\phi}_k = \left(1-\phi_k^2\right)\sum_{j=1}^{N} L_{jk} f\left(\phi_j - \phi_k\right), \tag{1}$$

where f is the coupling strength, which is an odd function, and L_{jk} is the connectivity matrix, i.e., $L_{jk} = 0$ if agents j and k are not connected, and $L_{jk} = 1$ if they are connected. Naturally $L_{kk} = 0$. The term $1-\phi_k^2$ is added to introduce natural fixed points in the model at values $\phi_k = \pm 1$ and to dampen the dynamics near these points, so that the values of the bias remain in the domain $\phi_k \in \left[-1,1\right]$.

At this point, our model lacks individuality of the agents, such as preference of the native language. Let us introduce the new term, describing the native language retention

$$\dot{\phi}_k = \left(1-\phi_k^2\right)\left[\sum_{j=1}^{N} L_{jk} f\left(\phi_j - \phi_k\right) + \gamma_k g\left(\eta_k - \phi_k\right)\right], \tag{2}$$

where $\eta_k = \pm 1$ is the native language parameter, and $g\left(\eta_k - \phi_k\right)$ is the retention function that controls the bias towards the native language of the individual agent. We assume that people have a natural tendency to lean towards their native language, due to various reasons, like cultural heritage, historical, philosophical, or political influence, etc. The parameter γ_k is the strength of individual agents' retention. Small values γ_k indicate that an agent is easily biased toward the other language, while large values indicate that the agent is deeply rooted towards its native language, e.g., "zealot" or "patriotism" parameter. It is obvious that the function g has to be an odd function as well.

Last, but not least, we have to consider the "status" of the language, described in [1]. The idea is that in reality, different languages have different perceived status of prestige. This factor appears for various natural reasons, such as the number of people speaking the language, the media influence, the access to information and education, etc. In our model, we introduce the influence term as

$$\dot{\phi}_k = \left(1-\phi_k^2\right)\left[\sum_{j=1}^{N} L_{jk} f\left(\phi_j - \phi_k\right) + \gamma_k g\left(\eta_k - \phi_k\right) + \beta_k h\left(P - \phi_k\right)\right], \tag{3}$$

where $P \in [+1,+1]$ is the prestige field parameter, and β_k is the susceptibility parameter of an individual agent. Small values β_k indicate that an agent is not easily influenced by an external influence, while large values indicate that the person is easily manipulated by an external influence, e.g., "zombie" parameter. Here h is also an odd function.

In our model, the bias of an agent depends on three factors: (i) the social network; (ii) the retention strength of the native language and its "patriotism" parameter; (iii) the prestige factor of the language, influenced by external sources and the "zombie" parameter of an agent.

3. Linear model

Although, our model is built for arbitrary functions f, g and h, in this paper we analyze only the linear case

$$\dot{\phi}_k = \left(1-\phi_k^2\right)\left[\sum_{j=1}^{N} L_{jk}\cdot\left(\phi_j-\phi_k\right)+\gamma_k\cdot\left(\eta_k-\phi_k\right)+\beta_k\cdot\left(P-\phi_k\right)\right]. \tag{4}$$

As we will see, even the simplest linear model has rich behavior. Another advantage of a linear model, is that its stability can be treated analytically.

3.1. Stability analysis

The non-trivial fixed points ϕ_k^* of the model (4) are found as solutions of

$$\sum_{j=1}^{N} L_{jk}\phi_j^* - \left(c_k+\beta_k+\gamma_k\right)\phi_k^* + \gamma_k\eta_k + \beta_k P = 0, \tag{5}$$

where ϕ_j^* are the fixed points of all the other equations for agents and $c_k = \sum_{j=1}^{N} L_{jk}$. Writing $\vec{\phi}^* = \left(\phi_1^*,\ldots,\phi_N^*\right)^T$ the equation (5) can be expressed in the vector form

$$A\vec{\phi}^* = \vec{b}, \tag{6}$$

where

$$b_k = \gamma_k\eta_k + \beta_k P,$$

$$A_{jk} = \delta_{jk}\left(c_k+\beta_k+\gamma_k\right) - L_{jk}, \tag{7}$$

where δ_{jk} is the Kronecker's delta. Now, the fixed points are obtained as

$$\vec{\phi}^* = A^{-1}\vec{b}. \tag{8}$$

To perform the linear stability analysis, we can write the biases as $\phi_k(t) = \phi_k^* + \varepsilon_k(t)$, where ε_k is a small perturbation, and leave only the linear terms in ε_k. This procedure yields

$$\frac{d\varepsilon_k}{dt} \approx \left(1-\left(\phi_k^*\right)^2\right)\left[\sum_{j=1}^{N} L_{jk}\cdot\varepsilon_j - \varepsilon_k\left(c_k+\beta_k+\gamma_k\right)\right], \tag{9}$$

or in the matrix for

$$\frac{d\vec{\varepsilon}}{dt} = -MA\vec{\varepsilon}, \tag{10}$$

where $\vec{\varepsilon} = \left(\varepsilon_1,\ldots,\varepsilon_N\right)^T$ and M is a diagonal matrix

$$diag(M) = 1-\left(\phi_k^*\right)^2. \tag{11}$$

In the case of symmetric coupling $L_{jk} = L_{kj}$, or an undirected network, the matrix is also symmetric and strictly diagonally dominant, since

$$A_{kk} = c_k + \beta_k + \gamma_k, A_{jk} = -L_{jk}, \Rightarrow |A_{kk}| \succ \sum_{j=1}^{N} |A_{jk}|. \quad (12)$$

Assuming that $L_{jk} \in [0,1]$ and $\beta_k \geq 0, \gamma_k \geq 0$, we can say that the matrix A is positive definite, which means that all its eigenvalues are real and positive. The diagonal matrix M is also positive, since by definition $|\phi_k| \leq 1$. Although we cannot say that $-MA$ it is symmetric, it is similar to a symmetric negative-definite matrix. This means that all the eigenvalues of the system's Jacobian are real and negative, which implies global stability of the fixed points $\vec{\phi}^*$.

For the directed network case $L_{jk} \neq L_{kj}$, we cannot guarantee that all the eigenvalues are real and negative, and more complicated dynamics might arise. The special cases are the fixed points $\phi_k^* = \pm 1$. In that case $\dot{\varepsilon}_k = 0$, which indicates the marginal stability of these points.

3.2. Network topologies

It is obvious that our model crucially depends on the topology of the network connections L_{jk}. In general, the elements of the matrix L_{jk} can have arbitrary values, not only 0 or 1, and can describe the connection strength between agents j and k. In this work, we consider only the undirected network cases, meaning that the connection matrix is symmetric. It is, however, obvious that for arbitrary network topologies, whether the matrix L_{jk} is symmetric or not, its diagonal elements should always be zero. As a measure of the entire network behavior, we will use the mean field bias $\langle \phi(t) \rangle$ defined as

$$\langle \phi(t) \rangle = \frac{1}{N} \sum_{j=1}^{N} \phi_j(t), \quad (13)$$

and its standard deviation. Here, we consider a few models that mimic certain real-life scenarios.

3.2.1. Fully connected network

The fully connected network, when all agents are identically connected to all the other agents, is the simplest connection topology. This simplicity is useful for analytical predictions and benchmarks, despite being quite unrealistic. However, a fully connected network can be reasonably accurate to model small communities, when everyone knows each other, or specific corporate networks, when everyone is connected to the same internal communication network. Usually, such networks produce a very coherent state when all the agents are aligned together. In such cases, it can be interesting to influence the mean field of the system by an external field, to see if it is possible to drive the mean field to an opposite polarity.

3.2.2. Small-world network

A small-world network, as introduced by Watts and Strogatz [22], interpolates between regular lattices and random graphs by introducing a small probability of long-range rewiring. In the context of our work, this topology captures the balance between local clustering, representing tightly connected communities, and occasional long-range interactions, such as those enabled by modern communication or migration. The small-world structure is particularly relevant for modeling realistic social systems, where individuals tend to interact more frequently within close groups but still maintain weak ties across the broader population. This heterogeneity can give rise to rich dynamical phenomena, including the formation of linguistic clusters, polarization, or partial synchronization.

4. Results and discussion

Let us now consider a few typical scenarios that we could model using (4). One of the standard scenarios is the language death, when the entire population eventually starts to strongly prefer only one of the languages. To achieve this scenario, we can construct a simple fully connected network of identical agents, meaning that they have the same native language η_k, and the same susceptibility to the prestige field β_k. Setting the prestige field P to an opposite language, we create a strong influence on the system.

To analyze the final state, we can draw certain thresholds for bias variables that mark danger zones for a language. This means that when the entire population's bias enters this zone, with a strong preference for a specific language, this puts the other language into danger of extinction.

In Figure 1, we have shown a simulation of the dynamics of a mean field $\langle \phi_k \rangle$ of $N = 100$ agents for different values of γ.

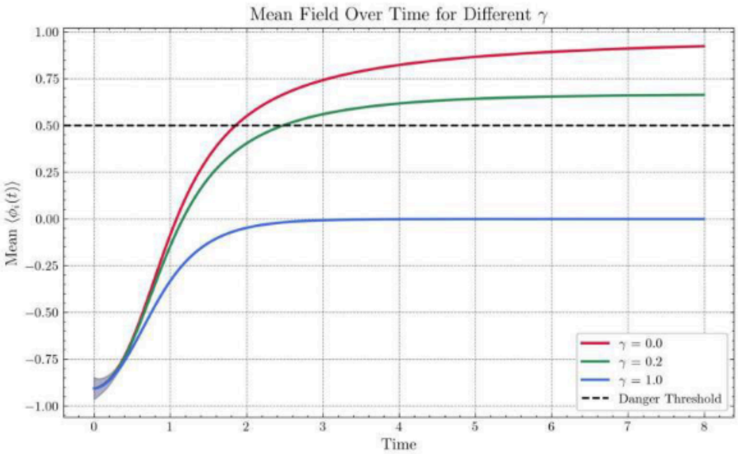

Fig. 1. The dynamics of the mean field $\langle \phi_k \rangle$ over time for different values of the parameter γ.
The simulation is performed for a fully connected network of identical agents, with native language parameters $\eta_k = -1$ and prestige susceptibility $\beta_k = 1.0$. The prestige field is set to $P = +1$, while the initial states of the agents are uniformly distributed within $\phi_k = U(-0.8, -1.0)$.

We can see that initially, the bias of all agents is strongly in favor of the native language. Exposed to an external prestige field P, their bias eventually leans towards the opposed language. We can see that if the patriotism parameter γ_k is small, the original preference for the native language can be overcome, putting it in danger of extinction. However, the strong patriotism ($\gamma_k = 1.0$ in the plot) can result in a bilingual outcome.

In the next scenario, we model a system initially localized at zero (bilingual bias), but eventually split the population into two distinct groups (Figure 2). To obtain such a state, we configured a system with half the population with a native language $A(\eta_k = +1)$, while the other half with a native language $B(\eta_k = -1)$.

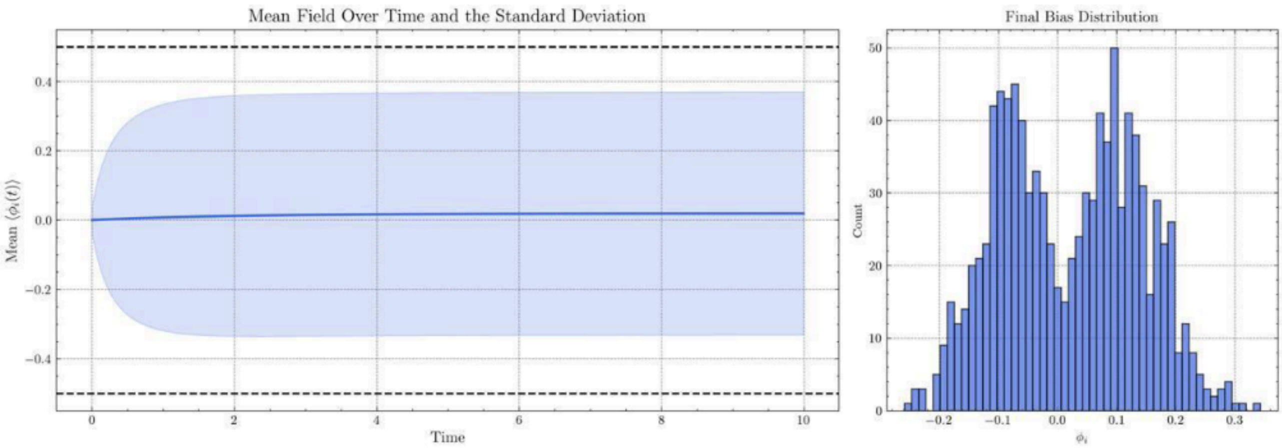

Fig. 2. The simulation of the linear system for $N = 1000$ agents. On the left, we can see the evolution of the mean field (solid line), and although it stays almost unchanged, the standard deviation (shaded area) becomes very large. On the right, there is a final distribution of the biases, with clear two separate peaks.

If the initial state of the system is strongly bilingual, then it generally tends to stay that way. In order to separate the population, we need to tweak the parameters in a specific way. First, in order to diminish the external influence, we set the zero-prestige field $P = 0$, and a very small susceptibility $\beta_k = 0.01$. Next, we set the entire population to be very patriotic, with a high value of $\gamma_k = 3.0$. Finally, the network topology was chosen to be a Watts-Strogatz small-world network.

5. Conclusion

In this work, we introduced a dynamical systems model to describe language bias evolution in populations embedded in social networks. By treating linguistic preference as a continuous variable and incorporating native identity retention, prestige influence, and agents' interactions, we developed a framework capable of capturing a wide range of realistic language dynamics scenarios.

Analytical results from the linearized model provide insight into the stability of fixed points. Numerical simulations on fully connected and small-world networks further illustrate how the interplay between topology and parameter heterogeneity governs the long-term outcomes.

This model not only advances the mathematical treatment of language competition but also exemplifies how methods from applied physics, particularly those related to networked systems and dynamical stability, can be effectively applied to social phenomena. Future work may extend this framework to incorporate dynamic networks, agent mobility, or feedback between bias and network structure, offering even deeper integration with techniques from complex systems and control theory.

References

1 Abrams D.M.,Strogatz S.H. (2003) Modelling the dynamics of language death. *Nature*, 424, 900. https://doi.org/10.1038/424900a

2 Castelló X., Eguíluz V.M., Miguel M.S. (2006) Ordering dynamics with two non-excluding options: Bilingualism in language competition. *New Journal of Physics,* 8, 308. https://doi:10.1088/1367-2630/8/12/308

3 Patriarca M., Leppänen T. (2004) Modeling language competition. *Physica A,* 338, 296. https://doi.org/10.1016/j.physa.2004.02.056

4 Kandler A., Steele J. (2008) Ecological models of language competition. *Biological Theory,* 3, 164. https://doi.org/10.1162/biot.2008.3.2.164

5 Mira J., Paredes A. (2005) Interlinguistic similarity and language death dynamics. *Europhysics Letters*, 69, 6. https://doi.org/10.1209/EPL/I2004-10438-4

6 Fagioli S., Radici E. (2021) Opinion formation systems via deterministic particles approximation. *Kinetic and Related Models, 14, 1.* https://doi:10.3934/krm.2020048

7 Sperlich S., Uriarte J. (2018) The Economics of Minority Language Use: Theory and Empirical Evidence for a Language Game Model. *SSRN Electronic Journal,* 1. https://doi:10.2139/ssrn.3445345

8 Marchetti G., Patriarca M., Heinsalu E. (2020) A bird's-eye view of naming game dynamics: From trait competition to Bayesian inference. *Chaos*, 30, 6, 063119. https://doi.org/10.1063/5.0009569

9 Marchetti G., Patriarca M., Heinsalu E. (2020) A Bayesian Approach to the Naming Game Model. *Front. Phys.*, 8, 00010. https://doi:10.3389/fphy.2020.00010

10 Marchetti G., Patriarca M., Heinsalu E. (2021) The role of bilinguals in the Bayesian naming game. *Physica D*, 428, 133062. https://doi.org/10.1016/j.physd.2021.133062

11 Mudd K., De Vos, C., De Boer, B. (2020) An agent-based model of sign language persistence informed by real-world data. *Language Dynamics and Change*, *10*(2), 158-187. https://doi.org/10.1163/22105832-bja10010

12 Charalambous C., Sanchez D., Toral R. (2023) Language dynamics within adaptive networks: an agent-based approach of nodes and links coevolution. *Frontiers in Complex Systems,* 1. https://doi.org/10.3389/fcpxs.2023.1304448

13 Gao Y, Liu W (2023) Measures to sustain endangered languages: A bilingual competition model with sliding mode control. *PLoS ONE*, 18(6): e0287850. https://doi.org/10.1371/journal.pone.0287850

14 Boissonneault M., Vogt P. (2021) A systematic and interdisciplinary review of mathematical models of language competition. *Humanit Soc Sci Commu,* 8, 21. https://doi.org/10.1057/s41599-020-00683-9

15 Soriano-Panos D., Lotero L., Arenas A., Gómez-Gardeñes J. (2018) Spreading Processes in Multiplex Metapopulations Containing Different Mobility Networks. *Phys. Rev. X*, 8, 031039. https://doi.org/10.1103/PhysRevX.8.031039

16 Zhanabaev Z.Z., Kozhagulov Y.T., Zhexebay D.M. (2016) FPGA implementations of scale-invariant models of neural networks. *Turkish Journal of Electrical Engineering and Computer Sciences,* 24, 6. doi:10.3906/elk-1504-204

17 Ibraimov M., Tynymbayev S., Skabylov A., Kozhagulov Y., Zhexebay D. (2022) Development and design of an FPGA-based encoder for NPN. *Cogent Engineering*, 9, 2008847. https://doi.org/10.1080/23311916.2021.2008847

18 Ussipov N., Akhtanov S., Zhanabaev Z., Turlykozhayeva D., Karibayev B., Namazbayev T., Almen D., Akhmetali A., Tang X. (2024) Automatic modulation classification for MIMO system based on the mutual information feature extraction. *IEEE Access,* 12, 68463 – 68470. doi:10.1109/access.2024.3400448

19 Turlykozhayeva D.A., Akhtanov S.N., Baigaliyeva A.N., Temesheva S.A., Zhexebay D.M., Zaidyn M., Ussipov N.M., Skabylov A.A. (2024) Evaluating routing algorithms across different wireless mesh network topologies using NS-3 simulator. *Eurasian Physical Technical Journal,* 21, 2(48). https://doi.org/10.31489/2024No2/70-82

20 Turlykozhayeva D., Temesheva S., Ussipov N., Bolysbay A., Akhmetali A., Akhtanov S., Tang X. (2024) Experimental performance comparison of proactive routing protocols in wireless mesh network using Raspberry Pi 4. *Telecom*, 5(4), 1008-1020. https://doi.org/10.3390/telecom5040051.

21 Ibraimov M.K., Kozhagulov Y.T., Zhexebay D.M., Sarmanbetov S.A. (2023) Implementation of functional block radio unit based on system-on-chip. *Eurasian Physical Technical Journal*, 20(4), 74–80. https://doi.org/10.31489/2023No4/74-80

22 Watts D., Strogatz S. (1998) Collective dynamics of 'small-world' networks. *Nature.* 393, 440–442. https://doi.org/10.1038/30918

AUTHORS' INFORMATION

Baibolatov, Yernur - PhD, Senior lecturer, al-Farabi Kazakh National University, Faculty of Physics and Technology, Department of Electronics and Astrophysics, Almaty, Kazakhstan; SCOPUS Author ID 35145187200; https://orcid.org/0009-0007-4125-898X, baibolatov.yernur@kaznu.kz

Yerkebulan Nalibayev - PhD, senior lecturer, al-Farabi Kazakh National University, Faculty of Physics and Technology, Department of Electronics and Astrophysics, Almaty, Kazakhstan; SCOPUS Author ID 55785834100, https://orcid.org/0000-0003-2217-4549; ednalibaev@gmail.com

Yeldos Kozhagulov - PhD, senior lecturer, al-Farabi Kazakh National University, Faculty of Physics and Technology, Department of Electronics and Astrophysics, Almaty, Kazakhstan; SCOPUS Author ID 57192878535, https://orcid.org/0000-0001-5714-832X; eldos.kozhagulov@kaznu.kz