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NUMERICAL ANALYSIS AND CHAOS CONTROL: A STUDY OF LORENTZ SYSTEMS WITH VISUAL BASIC FOR APPLICATION IMPLEMENTATION

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Abstract. *The study focuses on the numerical analysis and chaos control of Lorenz systems, leveraging Visual Basic for Application in Microsoft Excel for modeling and visualization. Chaotic systems, including the Lorenz attractor, represent a fundamental concept in nonlinear dynamics and chaos theory, characterized by sensitivity to initial conditions, nonlinearity, and fractal dimensionality. These properties make such systems valuable for analyzing complex processes in physics, biology, engineering, and economics. The research extends traditional exploration of the Lorenz attractor by introducing numerical methods such as the four-point Adams method with adaptive step selection. Classical parameter sets and non-classical modifications are examined. Additionally, a modified Lorenz system incorporating a supplementary term is analyzed, demonstrating distinct dynamic behaviors and trajectories. This work highlights the applicability of the developed Visual Basic for Application-based tools for solving nonlinear differential equations and visualizing complex attractors. The integration of Adams and Krylov methods enhances computational efficiency and precision. The outcomes align with previous studies and suggest that the software can address a wide range of applied mathematical and engineering challenges, including chaos management in dynamic systems. The findings underline the potential of the Lorenz attractor as a testbed for chaos control methods and numerical analysis techniques, with broader implications for scientific and practical applications across various disciplines.*

Keywords: Lorenz attractor, chaos theory, nonlinear dynamics, numerical analysis, Visual Basic for Application, Adams method, Krylov method, chaos control, dynamic systems.

1. Introduction

Chaos theory has become an important branch of modern science, bridging mathematics, physics, climatology and other disciplines. One of the key concepts is the Lorentz strange attractor, first described by Edward Lorentz in 1963. His work [1] became the basis for the study of dynamical systems where deterministic equations lead to complex and seemingly random behavior. This theory is of great importance, from understanding weather changes, to creating complex models in engineering and economics [2]. A strange Lorenz attractor is a mathematical set characterized by fractional dimensionality, aperiodicity, and sensitivity to initial conditions [3]. Studies have shown that the systems described by these equations are predictable in the short term and chaotic over large time intervals. This became the basis for new methods of numerical modeling, including the use of modern tools such as Visual Basic for Application (VBA) in Microsoft Excel [4]. In modern science, the use of the term “chaos” is associated with the need to describe systems that are characterized by outwardly completely random behavior and at the same time have a hidden order. The extremely urgent scientific problem of chaos management has not been solved yet. The study of various methods and laws of suppression of irregular oscillations in nonlinear systems with chaotic dynamics can be mentioned as the most important among the many existing aspects of its solution [5, 6]. The problem of controlling nonlinear objects and processes with chaotic dynamics is of great applied importance. This paper extends the traditional overview of the Lorentz attractor through numerical analysis, application of VBA for modeling, and discussion of possible approaches to chaos control.

2. Chaos Theory and Nonlinear Dynamics

Chaos theory studies deterministic dynamical systems whose behavior is sensitive to initial conditions, resulting in phenomena that appear random despite the strict determinism of the system. Key properties of chaos include:

– Sensitivity to initial conditions: even minimal changes in the starting data can significantly alter the evolution of the system [1].

– Nonlinearity: in chaotic systems there is a nonlinear dependence between variables, which leads to complex interactions [7].

– Fractional dimensionality: chaotic attractors tend to have a fractal structure [8].

These properties make chaos theory a fundamental tool for analyzing complex systems in various fields such as physics, biology, economics and engineering. Nonlinear dynamics, being the foundation of chaos theory, focuses on the study of the behavior of systems described by nonlinear differential equations, including phenomena such as bifurcations, aperiodic oscillations and fractal structures. Scientific developments in this field have contributed to the understanding of chaos mechanisms and the development of methods for its suppression in engineering and applied problems.

3. Lorenz Model

In 1963, a paper was published in the journal [1] that set the foundations of chaos theory. In this paper, a nonlinear autonomous system of ordinary differential equations of the first order (dynamical system) describing the motion of air flows in a flat fluid layer of constant thickness was first obtained from the system of Navier Stokes equations by decomposing the flow velocity and temperature into Fourier series with subsequent truncation to the first and second harmonics:

$$\begin{aligned} \dot{x} &= s(y - x), \\ \dot{y} &= rx - y - xz, \\ \dot{z} &= xy - bz, \end{aligned} \tag{1}$$

Where s, r and b – are some positive numbers, the parameters of the system.

Usually, studies of the Lorentz system are carried out at s =10, r =28 and b = 8/3 (classical values of the parameters). In general, chaos theory is a section of mathematics that studies the behavior of deterministic dynamical systems, where the solutions have a rather complex structure, so it seems that they behave randomly in time [9]. The dynamical system of differential equations (1) can also arise in the description of other processes [10-11]. For any solution of the Lorentz system, there exists a moment of time when the corresponding phase trajectory is immersed in a sphere of fixed radius. Therefore, there exists a limit set - Lorentz attractors - to which all trajectories of the dynamical system are attracted at $t \to \infty$ [12]. Thus, an attractor determines the behavior of solutions to the system of equations (1) over large time intervals. Due to the lack of exact methods for solving nonlinear dynamical systems of differential equations of general form, numerical methods such as, for example, the improved Euler scheme or the Runge-Kutta method are usually used to analyze the attractor structure.

4. Modeling the Dynamics of the Lorentz System in VBA Microsoft Excel

In our study, numerical integration of the differential equations describing the Lorenz attractor was performed using a program implementing the four-point Adams method with automatic step selection. The results of numerical integration are presented in Figs. 1-3. Figures 1-3 show the projections of the trajectories describing the strange Lorentz attractor on the yx, zx, yz planes, respectively.

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Figures 1-3 demonstrate graphs obtained as a result of numerical integration of the equations describing the Lorentz attractor with classical parameters. Figures 1–3 demonstrate phase trajectories of the classical Lorenz system with parameters s=10, r=28, b=8/3. All projections clearly show the characteristic two-petal structure of the attractor, reflecting the chaotic behavior of the system. The trajectories demonstrate strong sensitivity to the initial conditions: with a slight change in the initial coordinates, significantly different paths are formed in the phase space. This confirms the presence of a strange attractor and a complex topology of the phase region.

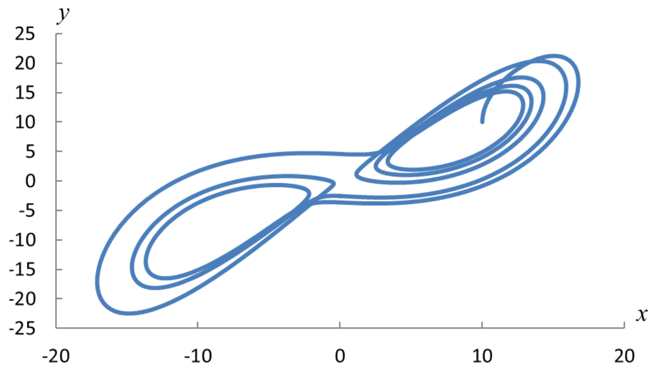

Fig. 1. Graph of the projection of the Lorentz attractor trajectories on the *yx* plane.

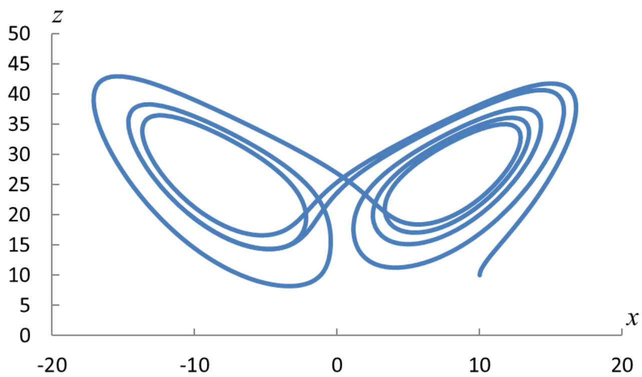

Fig. 2. Graph of the projection of the Lorentz attractor trajectories on the *zx* plane.

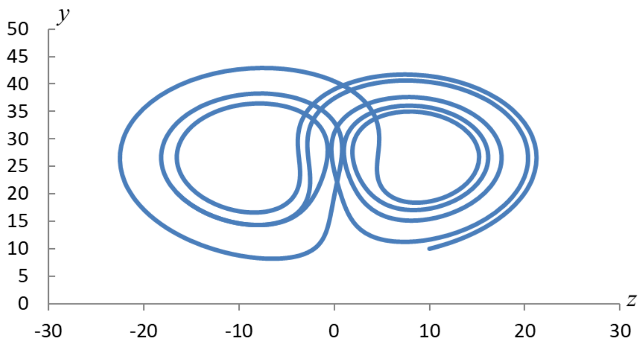

Fig. 3. Graph of the projection of the Lorentz attractor trajectories on the *yz* plane.

Figures 4–6 are phase projections of the trajectories of the Lorenz dynamic system with parameters s=15, r=35, b=10/3 which differ from the classical values and demonstrate an increased intensity of chaotic modes. The numerical experiment is based on the Adams method with automatic step selection, which ensures adequate integration accuracy while maintaining computational stability even under conditions of pronounced sensitivity to the initial conditions. Figure 4 (projection onto the yx plane) demonstrates a more pronounced stratification of the phase trajectories compared to the classical configuration. The attractor structure retains its characteristic two-petal shape, but asymmetry and an increase in the oscillation amplitude are observed. This indicates a shift towards more pronounced nonlinear effects, accompanied by an increase in the fractal dimension and an increase in the effect of sensitivity to the initial state of the system.

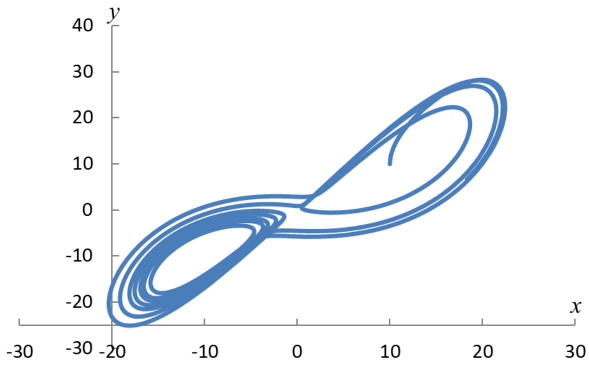

Fig. 4. Graph of the projection of the Lorentz attractor trajectories on the *yx* plane when s =15, r =35 and b =10/3.

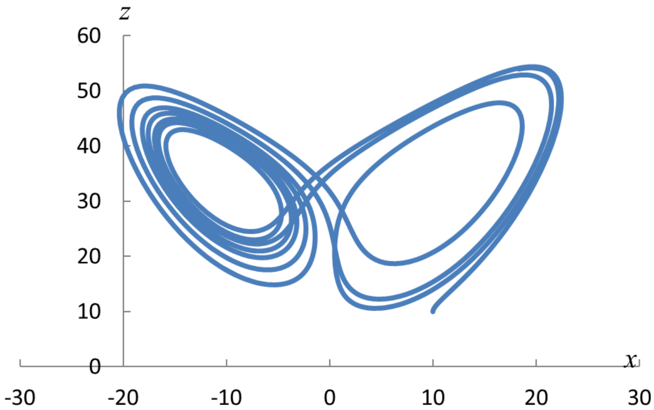

Fig. 5. Graph of the projection of the Lorentz attractor trajectories on the *zx* plane when s =15, r =35 and b =10/3.

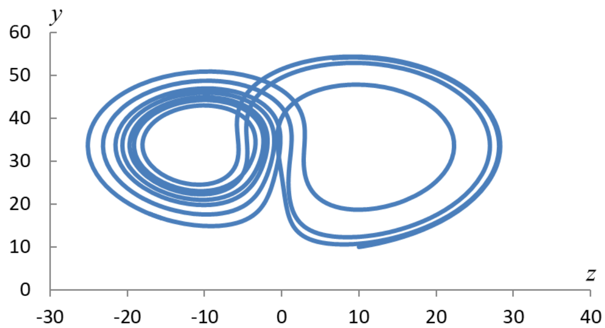

Fig. 6. Graph of the projection of the Lorentz attractor trajectories on the *yz* plane when s =15, r =35 and b =10/3

Figure 5 (projection onto the zx plane) clearly illustrates the spatial decomposition of the phase volume with the dominance of axially oriented spiral formations. It should be noted that there is a shortcoming in the notation: the z and x axes are not labeled, which requires correction for an accurate interpretation of the phase dynamics. Nevertheless, a qualitative analysis of the shape of the trajectories allows us to conclude that there is a pronounced spatial correlation between the variables for the specified set of parameters.

Figure 6 (projection onto the yz plane) emphasizes the presence of dynamic anisotropy in the system: the density of trajectories varies depending on the coordinates, which indicates the existence of quasi-stable regions of the phase space. The shape of the attractor acquires a more elongated structure, characteristic of systems with enhanced nonlinearity and a variable energy level.

Comparison with previously published studies, particularly [13] and [14], demonstrates that even when utilizing a non-standard computational environment-namely, VBA within Microsoft Excel-and operating under non-classical parameter regimes, our numerical approach maintains a high degree of agreement with established results. This consistency not only affirms the correctness of our implementation but also

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highlights the robustness and applicability of the method for investigating complex nonlinear dynamical systems.

While prior studies [13, 14] introduced modified forms of the Lorenz system and analyzed their chaotic behavior, fixed points, and stability characteristics using conventional high-level computational platforms such as MAPLE and MATLAB, the present work extends this line of research along a novel trajectory. Specifically, we propose a new modification of the Lorenz equations by introducing an additional nonlinear term into the first equation, which leads to previously undocumented dynamical regimes.

In our formulation, the modified Lorenz system is defined as follows:

$$\begin{aligned} \dot{x} &= s(y - x) + yz, \\ \dot{y} &= rx - y - xz, \\ \dot{z} &= xy - bz, \end{aligned} \tag{2}$$

where *s,r* and b are positive parameters, and the term *yz* introduced an additional nonlinear coupling into the evolution of x.

The results of numerical integration for this modified system are presented in Figure 7-9. These figures illustrate the significant changes in the topology of the phase space: the attractors become more elongated, asymmetric, and exhibit increased structural complexity. These qualitative transformations indicate the emergence of new dynamic modes and validate the sensitivity of the system to structural perturbations. The transition to these new behaviors is clearly visible across all phase projections, thereby confirming both the effectiveness of the proposed numerical method and the adequacy of the visualization framework implemented within the VBA environment.

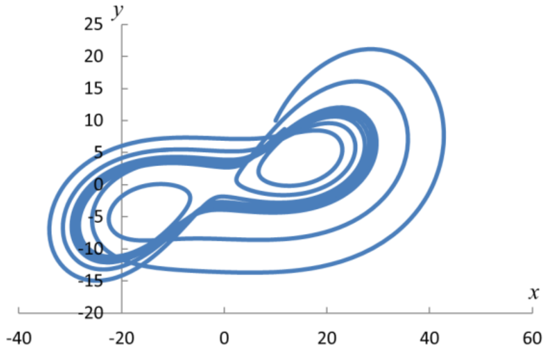

Fig. 7. Graph of the projection of the trajectories of the modified Lorentz attractor on the *yx* planes.

Based on the classical Lorenz model, the study demonstrates that the proposed methods and approaches lead to results similar to those presented in [13]. Numerical integration of the system of Lorentz differential equations was performed using a computer program written in VBA language and implementing the four-point Adams method with automatic selection of the integration step. The Krylov method of successive convergences was used to find the acceleration points for the Adams method. A VBA program implements both the Adams method and the Krylov method, ensuring that they work together.

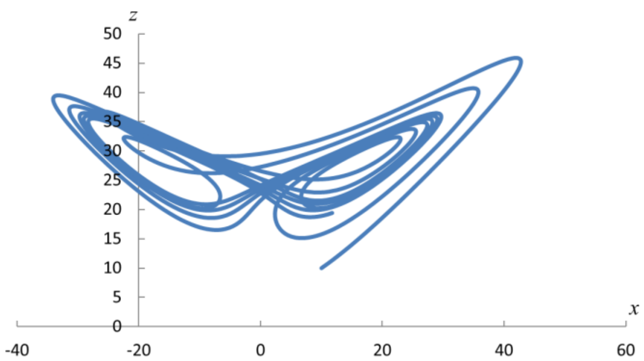

Fig. 8. Graph of the projection of the trajectories of the modified Lorentz attractor on the *zx* planes.

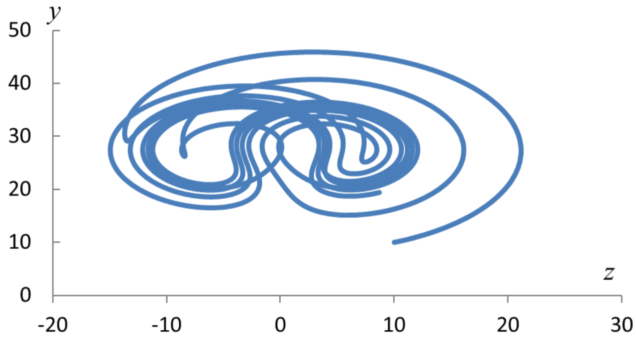

Fig. 9. Graph of the projection of the trajectories of the modified Lorentz attractor on the *yz* planes.

Typically, acceleration points are defined in regions where the field is weak and trajectories are nearly straight. These points are calculated using the Krylov method and are subsequently used to implement the Adams method. When numerically integrating a system of differential equations by the Adams method, the integration step is automatically selected so as to ensure a given accuracy of calculations [15-17].

5. Conclusion

In this study, a numerical model for analyzing the chaotic dynamics of the Lorenz system was implemented and tested using the Adams and Krylov methods integrated into a VBA implementation in Microsoft Excel. The stated goal — studying the behavior of the Lorenz system and developing tools for controlling its chaotic dynamics — was achieved through numerical modeling and comparative analysis of various system parameters, including both classical and modified configurations. The Adams method with automatic step selection allowed us to obtain stable numerical solutions with high accuracy, and the implementation of the Krylov method contributed to the acceleration of convergence in areas of the phase space with low dynamics. This is especially important in long-term modeling of complex nonlinear systems, where the accumulation of numerical errors can significantly affect the result. The phase trajectory graphs obtained for various sets of parameters demonstrated characteristic signs of chaotic behavior and confirmed the high sensitivity of the model to the initial conditions, which corresponds to theoretical expectations.

Comparison of the results with published works, where Maple and MATLAB were used, showed the coincidence of phase portraits, which confirms the correctness of the selected numerical methods and their implementation in the VBA environment. Despite the use of relatively simple tools, the developed model demonstrated efficiency and can be used both for educational purposes and for applied research of nonlinear dynamic systems. Thus, the work contributes to the practical implementation of methods of numerical analysis and visualization of chaos, and also demonstrates the possibilities of controlling complex dynamics through the selection of parameters and the use of computational algorithms. A promising direction for further research is the introduction of additional mechanisms for controlling chaos, including feedback, as well as expanding the model to other types of nonlinear systems.

Conflict of interest statement
The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

CRediT author statement
Shugayeva T.: Investigation, Formal Analysis, Writing-Reviewing and Editing; **Spivak-Lavrov I.:** Conceptualization, Methodology, Supervision; **Amantaeva A.:** Software and Analysis. The final manuscript was read and approved by all authors.

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