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## ELLIPTICALLY POLARIZED LASER-ASSISTED ELASTIC ELECTRON-HYDROGEN ATOM COLLISION IN COULOMB POTENTIAL

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*The advancement of laser technology is causing the research field of optics to become more active, and with the help of advancement of technology, more detailed information can be obtained. The primary goal of this work is to calculate differential cross section by using a mathematical model in presence of coulomb potential and elliptically polarized beam with single photon absorption. The developed model shows the differential cross section increases with wavelength and decreases with electron energy with elliptically polarized beam. The differential cross section become maximum at 1.56 radian polarized angle and minimum at -1.56 radian polarized angle. The observation is based on 1.5eV laser photon energy, laser field intensity  $10^{14} \text{Wcm}^{-2}$ , polarized angle 1.56 radian angle, and electron energy 0 to 600eV. Using the born first approximation and the Volkov wave function, the developed equation is obtained. The numerically obtained differential crosssection in this work is approximately  $10^{-19} \text{m}^2$  to  $10^{-20} \text{m}^2$ , which is less than the differential cross section obtained by Flegel et al. (2013), which is approximately  $10^{-17} \text{m}^2$ .*

**Keywords:** elliptical polarization, differential cross-section, Born first approximation, Volkov wave function.

### Introduction

These days, the study of electron-atom collisions in the presence of a laser beam has gotten a lot of interest. This is due to various application in disciplines of science (such as plasma heating or driven fusion) as well as in nuclear collision field. Observation of multiphoton events has become possible due to the development of powerful and adjustable lasers with relatively low light field intensities. When compared to the problems associated with field-free electron-atom scattering, the theoretical analysis of electron-atom collisions in the presence of a laser field becomes extremely complex. The study of collision in laser field introduces certain new parameters, such as laser frequency, intensity, polarization, and influence collision interactions. In interaction laser field (Photons) act as a "third body" in the universe. The existence of photons acts as a "third body" in the collision, "dressing" the atomic states [1]. For simplicity, to study the differential cross section in presence of laser field, the energy of the target atom is neglected because it does not change states during phenomena of collision but the potential is not neglected because potential replicate the electron-atom collision interaction. The simple reaction with laser-assisted elastic electron-atom scattering is given by equation (1) as,

$$e_{q_i} + A_i \rightarrow e_{q_f} + l\hbar\omega + A_i, \quad (1)$$

where A is target atom remains in its ground state during the collision while the electron exchanges  $l$  quanta with photon field ( $l < 0$  for absorption and  $l > 0$  for emission) to change momenta from  $q_i$  to  $q_f = q_i + \hbar\omega$ . The scattering equation represented in equation (1) also known as a free-free transition. The Kroll-Watson theorem [2] is one of the oldest and most reliable theoretical results in multiphoton physics. This theorem also described scattering of an electron by a potential in the presence of a low frequency linearly polarized laser field. The majority of theoretical studies of electron scattering by atoms in an intense radiation field are based on perturbation theory [3]. Kroll and Watson's also contribute to perturbation theory using the soft photon approximation and numbers of researchers have recently working on laser-assisted scattering more detail [4, 5].

Instead of number of difficult to observe laser-assisted electron impact atomic excitation in the presence of a strong field some of researcher is active to develop a simple model to understand the laser-assisted collision [7, 8]. The electron-target system in laser-assisted collisions, known as simultaneous electron-photon excitation (SEPE), can absorb or emit one or more photons from the laser field, causing the atom to be excited. In laser-assisted elastic and inelastic [9,10] electron-atom collisions, the exchange of one or more photons between the electron-atom system and the laser field has been observed in several experiments. In the presence of a field, the collision can be treated in such a way that electron-field coupling is the dominant process. Mason and Newell in 1982 reported experimental evidence of atoms being excited by both electrons and photons at the same time. The majority of the experimental studies have been conducted with noble gases [11], with Abdelkader et al. [12] conducting a recent one with Nd: YAG laser. He-target and low-field laser field were tested in two states: (i) SEPE, electron energy below the excitation threshold of the metastable  $2S^3$  state collide with ground state  $1S^1$  and achieve excitation because laser supplies energy; and (ii) SEPE, electron energy above the excitation threshold of the metastable  $2S^3$  state collide with ground state  $11S$  and achieve excitation because laser supplies energy. (ii) SEPE has also been observed from higher excited states [13].Shinha et al. [4] investigated the free-free transition for an electron-hydrogen atom system in the ground state in the presence of an external homogeneous, monochromatic, and linearly polarized laser field at very low incident energies. The incident electron is thought to be non-perturbatively dressed by the laser field by selecting Volkov solutions in both the initial and final channels. For single-photon absorption or emission in the soft photon limit, the laser intensity is much lower than the atomic field intensity, and the laser-assisted differential and total elastic cross-sections are calculated.

The hydrogen atom is one of the most basic atoms to work with and can be used to collect interesting aspects of the problem. The free-free process can be studied theoretically on several levels. The goal of this paper is to investigate the impact of various collision and laser parameters on the collision process in an elliptically polarized laser-assisted elastic electron-hydrogen atom collision. The authors attempt to develop a detailed calculation of differential cross section for laser-assisted electron-hydrogen collisions in this paper. The Volkov wave [14] is used to treat the interaction between the field and the projectile as non-perturbation. The authors focused on the elliptical polarization laser field for polarized potential. To begin, consider a collision event in which an incoming electron with momentum  $k_i$  interacts with a hydrogen atom that is initially in the state  $I$  in the presence of a single-mode laser beam and moves to the excited state  $j$  through the exchange of  $l$  photons between the electron and the laser field.

## 1 Theory and Method of calculation

### 1.1 Volkov - wave function

Consider an elastic collision at the ground state between a fast (non-relativistic) electron of mass ( $m$ ) and charge ( $-e$ ) with hydrogen as the target atom. The collision takes place in the presence of a laser field, which is assumed to be a monochromatic, single-mode, and homogeneous electromagnetic field. Therefore, the vector potential of a field propagating along the  $Z$ -axis in the Coulomb gauge is obtained as,

$$A(t) = A_0 \left\{ \hat{x} \cos(\omega t) + \hat{y} \sin(\omega t) \tan\left(\frac{\eta}{2}\right) \right\}, \quad (2)$$

where  $A_0 = \frac{cE_0}{\omega}$ ,  $E_0$  is the electric field and  $\omega$  is frequency,  $\eta$  is measured the degrees of ellipticity of the field.  $\eta$  determine the nature of laser field as, for linear polarization ( $\eta = 0$ ), for circular polarization ( $\eta = \frac{\pi}{2}$ ) and for elliptical polarization ( $-\frac{\pi}{2} \leq \eta \leq \frac{\pi}{2}$ ). The wave function of the projectile embedded in the field is given by the non-relativistic Volkov wave function

$$\chi^V(\vec{r}, t) = (2\pi)^{-3/2} \exp\left(i\vec{k} \cdot \vec{r} - \frac{i}{\hbar} \int_{-\infty}^t \left(\frac{p^2}{2m} + \frac{e}{mc} \vec{A} \cdot \vec{p}\right) dt\right) \quad (3)$$

On solving integration of equation (3),

$$\chi^V(\vec{r}, t) = (2\pi)^{-3/2} \exp\left(i\vec{k} \cdot \vec{r} - \frac{iE_k t}{\hbar} - \frac{ie}{m\omega c} \vec{A}_0(t) \left[ (\hat{x} \cdot \vec{k}) \sin\omega t - (\hat{y} \cdot \vec{k}) \tan\left(\frac{\eta}{2}\right) \cos\omega t \right]\right) \quad (4)$$

Assuming,  $\hat{x} \cdot \vec{k} = R \cos\gamma_k$ ,  $(\hat{y} \cdot \vec{k}) \tan\left(\frac{\eta}{2}\right) = R \sin\gamma_k$  as  $R = \frac{ie}{m\omega c} \vec{A}_0(t)$  then equation (4) becomes

$$\chi^V(\vec{r}, t) = (2\pi)^{-3/2} \exp\left(i\vec{k} \cdot \vec{r} - \frac{iE_k t}{\hbar} - R \sin(\omega t - \gamma_k)\right) \quad (5)$$

Equation (5) is the required form of the Volkov wave function with  $R = \alpha_0 D_0 k$ ,  $D_0 = [\cos^2 \theta + \sin 2\theta \tan 2\eta 21/2, \tan \gamma k = k.yk.x \tan \eta 2$  and  $\alpha 0 = eE0m\omega 2$ .

### 1.2 Calculation of S-matrix element

The general S-matrix theory is used to study multi-photon ionization and also define T-matrix in terms of S-matrix matrix elements, i.e. transition amplitude is the S-matrix matrix elements. Kroll and Watson in 1973 developed the S-matrix formulation for a low-frequency response. This formulation was developed in the search for a non-perturbative approach hand suitable not only for free-free scattering but also for ionization, recombination, and excitation. Unlike other methods, this method does not suffer from energy loss or gain. As a result, in the case of multi-photon ionization, this method is more applicable for calculating total transition rate and differential cross-section, because total transition rate divided by incident flux gives the differential cross-section, which is proportional to the square of the transition matrix. The S-matrix element is given by

$$S = \frac{-i}{\hbar} \langle \chi_{\vec{k}_f}^* V \chi_{\vec{k}_i} \rangle \tag{6}$$

Equation (6) related transition amplitude from the momentum state  $\vec{k}_i$  to  $\vec{k}_f$  as

$$S_{\vec{k}_f \vec{k}_i} = \frac{-i}{\hbar} \iint_{-\infty}^t \chi_{\vec{k}_f}^* V \chi_{\vec{k}_i} d^3 r dt \tag{7}$$

Where  $\vec{k}_i$  the initial is wave vector of the particle and  $\vec{k}_f$  is the final wave vector of the scattered particle. Substituting the value of for  $\chi_{\vec{k}_i}^*$  and  $\chi_{\vec{k}_f}$  from (5) in (7) and on solving,

$$S_{\vec{k}_f \vec{k}_i} = \frac{-i}{\hbar} \frac{1}{(2\pi)^3} \iint_{-\infty}^t \exp[-i(\vec{k}_f - \vec{k}_i) \cdot \vec{r}] \exp\left[\frac{(E_{k_f} - E_{k_i})t}{\hbar}\right] V(\vec{r}) \exp[i(\vec{k}_f - \vec{k}_i)\alpha_0 A_0 \sin(\omega t - \gamma_k)] d^3 r dt \tag{8}$$

Let us assume  $\Delta = \vec{k}_f - \vec{k}_i$  be the momentum transfer then equation (8) become

$$S_{\vec{k}_f \vec{k}_i} = \frac{-i}{\hbar} \hat{V}(\Delta) \int_{-\infty}^t e^{i(\Delta \cdot \vec{\alpha}_0) A_0 \sin(\omega t - \gamma_k)} e^{i(E_{k_f} - E_{k_i})\frac{t}{\hbar}} dt \tag{9}$$

where,  $\hat{V}(\Delta) = \frac{1}{(2\pi)^3} \int e^{-i\Delta \cdot \vec{r}} V(\vec{r}) d^3 r$  and  $V(\vec{r})$  is independent of time (t) and authors considering space integration. So that  $\hat{V}(\Delta)$  can be taken outside the time integration. On using the generating function of the Bessel Polynomial is [15],

$$e^{ix \sin \phi} = \sum_{-\infty}^{\infty} J_n(x) e^{ni\phi} \tag{11}$$

where,  $x = \Delta \cdot \alpha_0 D_0$  and  $\phi = \omega t$  equation (11) becomes

$$e^{i\Delta \cdot \alpha_0 A_0 \sin \omega t} = \sum_{l=-\infty}^{\infty} J_l(\Delta \cdot D_0 \alpha_0) e^{il\omega t} \tag{12}$$

On substituting the value from equation (12) in equation (9) the S-matrix element becomes

$$S_{\vec{k}_f \vec{k}_i} = \frac{-i}{\hbar} \int_{-\infty}^t T_{\vec{k}_f \vec{k}_i}^l e^{i(E_{k_f} - E_{k_i} + l\hbar\omega)\frac{t}{\hbar}} dt \tag{13}$$

here  $T_{\vec{k}_f \vec{k}_i}^l = \hat{V}(\Delta) \sum_l J_l(\Delta \cdot D_0 \alpha_0) e^{il\gamma_k}$  is the transition matrix from the momentum state  $k_i$  to  $k_f$ , and time-independent. Therefore equation (13) becomes

$$S_{\vec{k}_f \vec{k}_i} = \frac{-i}{\hbar} T_{\vec{k}_f \vec{k}_i}^l \int_{-\infty}^t e^{i(E_{k_f} - E_{k_i} + l\hbar\omega)\frac{t}{\hbar}} dt \tag{14}$$

### 1.3 Calculation of transition matrix and differential cross section

The relation for the differential cross section of an electron with the transfer of l photon is

$$\frac{d\sigma}{d\Omega} = \frac{m^2}{(2\pi)^2 \hbar^4} \frac{k_f}{k_i} |T_{\vec{k}_f \vec{k}_i}^l|^2 \tag{15}$$

On substituting the value of  $T_{k_f k_i}^l$  and  $\widehat{V}(\Delta)$  from above in (15) we get a differential crosssection for  $l = 0$  (no photon transfer during scattering) as,

$$\frac{d\sigma}{d\Omega} = \frac{m^2}{(2\pi)^2 \hbar^4} \sum_l J_l^2(\Delta, D_0 \alpha_0) |\widehat{V}(\Delta)|^2 \quad (16)$$

Moreover, equation (16) can be represented as

$$\left(\frac{d\sigma}{d\Omega}\right)^{\text{free-free}} = \left(\frac{d\sigma}{d\Omega}\right)^{\text{field-free}} \sum_l J_l^2(\Delta, D_0 \alpha_0) \quad (17)$$

Using sum rule  $\sum_l J_l^2(\Delta, D_0 \alpha_0) = 1$  equation (17) becomes,

$$\left(\frac{d\sigma}{d\Omega}\right)^{\text{free-free}} = \left(\frac{d\sigma}{d\Omega}\right)^{\text{field-free}} \quad (18)$$

Thus, in this limit free-free differential cross section is equal to the diffrential crosssection in the absence of the laser field.As we have

$$|T_{k_f k_i}^l|^2 = \sum_l J_l^2(\Delta, D_0 \alpha_0) |\widehat{V}(\Delta)|^2 \quad (19)$$

For spherically symmetric potential,  $V(\vec{r}) = V(r)$  therefore we have,

$$\widehat{V}(\Delta) = -\frac{1}{2\pi^2} \int_0^\infty \frac{\sin(\Delta r)}{\Delta} V(r) dr \quad (20)$$

If we choose  $V(r)$  as coulomb potential,

$$V(r) = -\frac{ZZ'e^2}{r} \quad (21)$$

The Fourier transform of the coulomb potential of equation (21) is given by

$$\widehat{V}(\Delta) = \frac{e^2}{2\pi^2 \Delta^2} \quad (22)$$

On substituting the value of equation (22) in equation (19),

$$|\widehat{V}(\Delta)|^2 = \left| \frac{e^2}{2\pi^2 \Delta^2} \right|^2 = \frac{e^4}{4\pi^4 \Delta^4} \quad (23)$$

The higher-order terms can be neglected for small momentum transfer. To calculate  $\sum_l J_l(\Delta, D_0 \alpha_0)$  all we have to do is to replace  $x$  by  $\Delta, D_0, \alpha_0$  in the expression of Bessel function under  $l = 1$  that corresponds to stimulated Bremsstrahlung (one photon emission) at low frequency [16],

$$J_l(\Delta, D_0 \alpha_0) = \frac{\Delta D_0 \alpha_0}{2} \quad (24)$$

Also at low frequency

$$|T_{k_f k_i}^l|^2 = \sum_l J_l^2(\Delta, D_0 \alpha_0) |\widehat{V}(\Delta)|^2 = \frac{\Delta^2 D_0^2 \alpha_0^2}{4} \left( \frac{e^4}{4\pi^4 \Delta^4} \right) \quad (25)$$

On substituting the value of equation (25) in equation (19),

$$\frac{d\sigma}{d\Omega} = \frac{m^2 D_0^2 \alpha_0^2 e^4 k_f}{64\pi^6 \hbar^4 k_i \Delta^2} \quad (26)$$

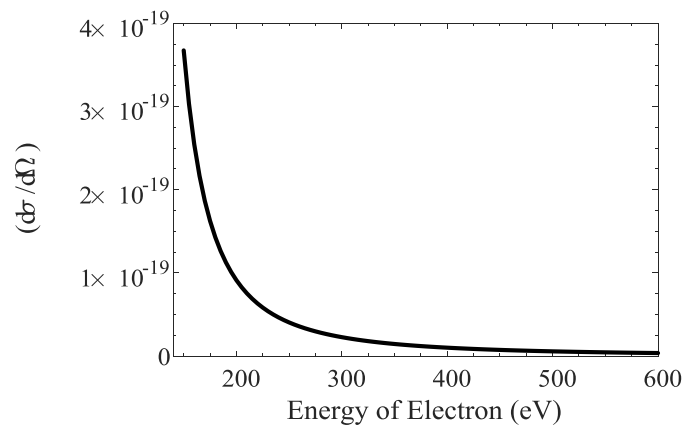
Substituting for  $\Delta^2 = k_i^2 \left[ \left(1 - \frac{\hbar\omega}{E_{k_i}}\right) - 2 \left(1 - \frac{\hbar\omega}{E_{k_i}}\right)^{\frac{1}{2}} \cos\theta + 1 \right]$  in equation (26) and solving the case of inverse Bremsstrahlung ( $l = -1$ ),

$$\frac{d\sigma}{d\Omega} = C \left( \cos^2\theta + \sin^2\theta \tan^2\frac{\eta}{2} \right)^{\frac{1}{2}} \frac{I\lambda^4}{E_{k_i}} \left( 1 + \frac{\hbar\omega}{E_{k_i}} \right)^{\frac{1}{2}} \left[ \left( 1 + \frac{\hbar\omega}{E_{k_i}} \right) - 2 \left( 1 + \frac{\hbar\omega}{E_{k_i}} \right)^{\frac{1}{2}} \cos\theta + 1 \right]^{-1} \quad (27)$$

Here,  $C$  is  $\frac{m^2 \alpha_0^2 e^4}{64\pi^6 \hbar^4} = \frac{1}{256\pi^9}$  in a.u.,  $D_0$  is  $(\cos^2\theta + \sin^2\theta (\tan\frac{\eta}{2})^2)^{1/2}$ ,  $m$  is mass of the electron,  $k_i$  is initial momentum vector of the electron,  $E_{k_i}$  is the initial kinetic energy of the incident electron,  $\hbar\omega$  is photon energy of the laser,  $l$  is no. of the photon transfer during the interaction,  $\theta$  is scattering angle,  $\Delta$  is momentum transfer,  $\alpha_0$  is  $\frac{eE_0}{m\omega^2}$ ,  $E_0$  is the amplitude of the electric field of the laser,  $E_0$  is  $\frac{8\pi I}{c} \eta$  is elliptically polarized angle and  $I$  is the intensity of the laser.

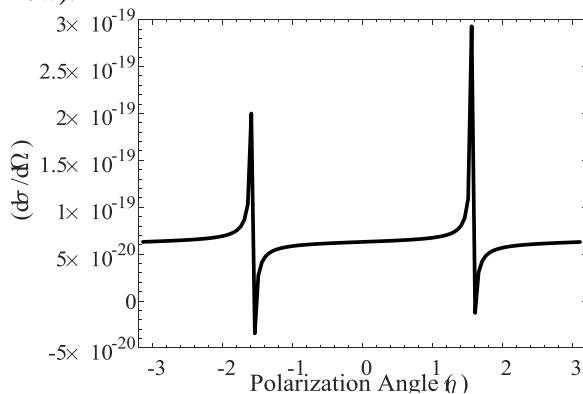
### 3 Results and Discussion

The authors investigate the elastic scattering of an electron-atom interaction with photon absorption in the presence of an elliptically polarized laser field in this paper. Equation (27), the new resultant equation, is used to investigate the differential cross section. The differential cross section is determined by, the wavelength of the assisting laser field, and the electron's kinetic energy. Figure 1 depicts the differential cross section with the electron beams energy. The representation is based on visible energy photon=1.5eV, high intensity laser field  $10^{14} \text{Wcm}^{-2}$  (frequency =  $10^{14} \text{Hz}$ ) Sprangle and Hafizi [17],  $\theta = 0.209$  radian,  $\eta = 1.56$  radian angle and energy of electron 0 to 600eV. The SI unit for differential cross section is  $\text{m}^2$ . Because the intensity of a laser is directly proportional to its differential cross section, and the differential cross section studied in this work is for visible photons at high intensities, As a result, for high intensity laser radiation, we consider the existence of the laser that Sprangle and Hafizi used in their work.

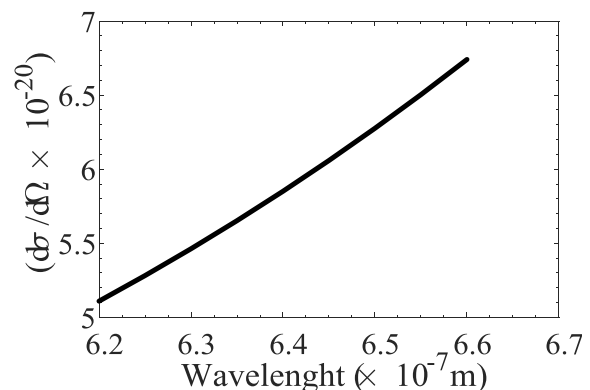


**Fig.1.** Variation of  $d\sigma/d\Omega$  with K.E. of the incident electron at  $\eta = 1.56$  radian and  $\theta = 2.09$  radian

At 25eV, the differential cross section is high, and the differential cross section is not observed below 25eV. This is due to the fact that the interaction of low-energy electrons with atoms does not require atom (probability of interaction-free and bounded electron is zero). The differential cross section decreases as the energy of the electron increases because the interaction between the incidence electron and the atom increases (probability of interaction of free and bounded electron is high). When the incidence electron's energy is greater than 500eV, the differential cross section is minimum and constant (probability of interaction-free and bounded electron very low). Figure 2 shows the variation of a differential cross section with an elliptical polarization angle of  $\theta = 2.09$  radian and energy of 10eV. The differential cross section assistant by the elliptically polarized angle is minimum at  $-1.56$  radian and maximum at  $+1.5$  radian. The differential cross section between  $-1.5$  radian and  $+1.56$  radian angle is nearly constant and decreases sharply at  $-1.56$  radian and  $+1.56$  radian. This sharp increase and decrease are due to the elliptical polarized potential and the non-uniform interaction between free and bounded electrons (probability of interaction is either high or low).



**Fig.2.** Variation of  $\frac{d\sigma}{d\Omega}$  with polarizing angle ( $\eta$ ).



**Fig.3.** Variation of  $\frac{d\sigma}{d\Omega}$  with assisting laser photon wavelength

As shown in Figure 3, the differential cross section increases with the wavelength of the laser photon at  $\eta = +1.56$  radian, electron energy 10eV. Because the energy of the laser photon assisting the electron decreases as the wavelength increases, the differential cross section increases. The reduction in photon energy benefits electrons by reducing the interaction of free electrons with bound electrons. Because the free electron was diverted away from the reference target, the differential cross section increased. The larger the differential cross section value, the less accurate the information about the target. The ellipticity of the laser field affects the angular distribution of scattered electrons in the simplest geometry. This is due to the fact that it destroys the axial symmetry of the angular distribution that exists for  $\eta = 0$  to the direction of the polarization vector.

## Conclusion

The developed equation (27) depicts the differential cross section for an elliptically polarized potential with single-photon absorption. The differential cross section is determined by the polarization angle, electron energy, laser intensity, and photon energy. The observed differential cross section is large at 25eV (electron energy), but not below 25eV. The differential cross section is minimum and constant at high energies greater than 500eV. The differential cross section increases as the wavelength of the laser photon assisting the electron increases. Furthermore, the ellipticity of the laser field influences the angular distribution of scattered electrons by destroying axial symmetry. The nature differential cross section is based on existence numerical data from Sprangle and Hafizi 2014 research work.

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## REFERENCES

- 1 Mason N.J. Laser-assisted electron-atom collisions. *Rep. Prog. Phys.* 1993, Vol. 56, pp.1275-1346.
- 2 Kroll N.M., Watson K.M. Charged-particle scattering in the presence of a strong electromagnetic wave. *Phys. Rev. A.* 1973, Vol.8, Is.2, pp.804-809.
- 3 Yadav K., Nakarmi J.J. Free-free scattering theory of the elastic scattering of an electron. *Int. J. Phys.* 2015, Vol.3, pp.32-39.
- 4 Sinha C., et al. Laser-assisted free-free transition in electron-atom collisions. *Phys. Rev. A.* 2011, Vol.83, Iss.6, pp.1-3.
- 5 Mittleman M.H. *Introduction to the theory of laser-atom interactions*. Springer Science and Business Media, Switzerland, 2013. 198 p.
- 6 Flegel A.V., et al., Control of atomic dynamics in laser-assisted electron-atom scattering through the driving-laser ellipticity. *Phys. Rev. A.* 2013, Vol. 87, Is.3, pp.1-5.
- 7 Mason N.J., Newell W.R. Simultaneous electron-photon excitation of the helium  $2^3S$  state. *J. Phys., B.* 1987, Vol.20, Is.10, pp.323-325.
- 8 Wallbank B., et al. Simultaneous off-shell excitation of He  $2^3S$  by an electron and one or more photons. *Z. Phys. D At. Mol. Clusters.* 1988, Vol.10, Is.4, pp.467-472.
- 9 Makhoute A., et al. Electron-impact elastic scattering of helium in the presence of a laser field: non perturbative approach. *J. Phys. B: At., Mol. Opt. Phys.* 2016, Vol.49, Is.7, pp.1-30.
- 10 Agueny H. et al. Laser-assisted inelastic scattering of electrons by helium atoms. *Phys. Rev. A.* 2015, Vol.92, Is.1, pp.1-5.
- 11 Makhoute A., Agueny H., Chqondi S. Floquet theory in electron-helium scattering in Nd: YAG laser field. *Opt. Photonics J.* 2013, Vol. 3, pp.18-27.
- 12 Wallbank B., et al. Simultaneous electron-photon excitation of He  $2^3S$ : an experimental investigation of the effects of laser intensity and polarisation. *J. Phys. B: At., Mol. Opt. Phys.* 1990, Vol.23, Is.17, pp.2997-3000.
- 13 Volkov D.M. The solution for wave equations for a spin-charged particle moving in a classical field. *Z. Phys.* 1935, Vol.94, pp.250-260.
- 14 Watson G.N. A treatise on the theory of Bessel functions. Cambridge University Press. (1995).
- 15 Yadav K. Theoretical study of multi-photons ionization of hydrogen atom by nonperturbative method with intense laser pulse, PhD Thesis, Tribhuvan University, Nepal 2016.
- 16 Flegel A.V. et al. Analytic description of elastic electron-atom scattering in an elliptically polarized laser field. *Phys. Rev. A.* 2013, Vol.87, Is.1, pp.2-6.
- 17 Sprangle P., Hafizi B. High-power, high-intensity laser propagation and interactions. *Phys. Plasmas.* 2014, Vol.21, pp.1-2.