DOI 10.31489/2022No3/84-90

UDC 621.396.49

CLUSTER ROUTER BASED ON ECCENTRICITY

Zhanabaev Z.Zh, Akhtanov S.N., Turlykozhayeva D.A.*, Ussipov N.M., Ibraimov M.K.

Department of Solid State Physics and Nonlinear Physics, Al-Farabi Kazakh National University, Almaty, Kazakhstan, turlykozhayeva.dana@kaznu.kz

In this paper, a cluster router based on eccentricity was worked out, related to the field of telecommunications, especially, to the field of message transmission. Messages in this router are transmitted as packets along the route specified in it between devices connected to the network. Each node in this network is assigned a unique address, thanks to which routing can be accelerated. Each router forms a routing map, thanks to the calculated eccentricity of nodes, with which the physical route of the packet is selected at the logical address of the cluster. In addition, the routing map is stored in the register and non-volatile memory of the device to prevent information loss. To analyze this cluster device, a fractal analysis of the UV-flower model network was carried out and the information dimensions of Tsallis and Renyi were calculated.

Keywords: cluster router, eccentricity, complex networks, box covering algorithms, Tsallis and Renyi dimensions.

Introduction

With the development of Internet technology and telecommunications, research is being actively carried out in the field of routing complex networks, which raises the question of developing effective routers [1-2]. In addition, the number of mobile device users is growing, which complicates the network and routing between subscribers. In addition, a large increase in traffic is observed in complex networks such as the Internet [3], as well as in various social networks. It is known that complex networks form a large number of links between subscriber nodes that cover a large physical area. Energy efficiency optimization and fast routing are among the most intelligent and relevant tasks today [4,5].

Routing protocols play an important role in the life and power consumption of sensor nodes. Since each node is powered by a battery and has a limited resource. An optimized routing protocol saves these node resources [6, 7]. There are many routing protocols for a wireless sensor network [8, 9]. The main ones are (1) flat [10], (2) location based [11] and (3) hierarchical [12]. With flat routing, each sensor node interacts with each other. Location-based routing information is only transmitted in a specific area. Particular attention is paid to routing protocols of a hierarchical type, which provide the best results in terms of energy efficiency, throughput and routing [4].

Cluster routing protocols divide a network into several clusters by determining the network eccentricity. Each cluster consists of one main cluster node and several nodes. Each node collects information and forwards it to the main node. Further, data is sent from the main node to the central pre-determined station. These operations are performed in order to cover the network with a minimum number of cells. To solve these problems, it is necessary to use box-covering algorithms, such as CIEA, MEMB, GC, etc. [13-18]. These algorithms are used to calculate the fractal dimension of large-scale networks by covering the network with the minimum possible number of network cells. The minimum number of cells allows you to use base stations less and find the shortest paths to nodes or cells faster. After the network has been covered by various algorithms, we analyzed this cluster router by calculating different information dimensions of the routing map [19, 20].

The classical dimension is fractal dimension that mainly focuses on the relationship between the number and the size of boxes. However, fractal dimension does not consider the information inside the box. To review the information in each box Wei at al. proposed a classical information dimension, where boxes with a large number of nodes have a maximum impact on the information dimension. However, in some cases, the boxes containing a small number of nodes are significant in the network. And moreover, classical information dimension cannot decide which box has greater influence on the fractal property. Considering these cases, Zhang et al. proposed the Tsallis information dimension based on Tsallis entropy [21, 22]. This is because entropy can focus on different information within the box. The Tsallis entropy is one of the general forms of the Shannon entropy [23, 24], which is controlled by the parameter q. It should be noted that Tsallis entropy is currently used in many areas of human activity, for example, in the analysis of medical images, stellar polytropes, community detection and physics of the cardiovascular system. Closely related to Tsallis entropy is the Renyi entropy, which is considered as the non-extensive statistical mechanics. Based on Renyi entropy, Renyi information dimension was proposed by Duan et al. In Renyi information dimension, the presence of the parameter α makes the proposed method more flexible and expands the possibilities of its use in many areas. For example, we can realize the importance of Renyi entropy in ecology and statistics as index of diversity. In addition, the Renyi entropy is important in quantum information, where it is used as a measure of entanglement.

In this work we proposed a new cluster router based on CIEA, which can divide the network into clusters. Then we calculated the Renyi and Tsallis information dimensions by partitioning the network into boxes. The use of various box covering algorithms for calculating the information dimension has been insufficiently explored in scientific sources. In addition, the CIEA has not previously been used in the calculations of the Renyi and Tsallis information dimensions [25-27].

1. Cluster router based on eccentricity

The cluster router based on eccentricity is equipped with a controller with a switching matrix, bidirectional ports for connection to a control machine and a programmable logic integrated circuit (FPGA) with non-volatile memory, made with the possibility of routing through multiple subnets. Figure 1 shows a block diagram of a cluster router. The cluster router contains a controller with a switching matrix 1, bidirectional ports 2 for connection to a control automaton 3 and a programmable logic integrated circuit (FPGA) 4 with non-volatile memory 5.

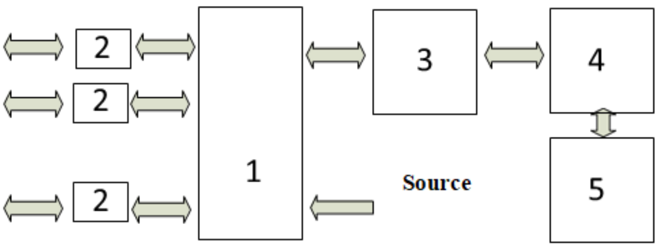


Fig. 1. Block diagram of cluster routing based on eccentricity.

Well-known routers build a route across the entire network and are based mainly on Dijkstra's algorithm. However, these routers are not effective because they contain a large number of network nodes. In contrast, a cluster router divides the network into clusters by calculating the eccentricity, which reduces the number of operations to find the shortest path in the network topology.

To describe a clustered router, we denote the network cluster size as l_b and the cluster radius as r_b, where $l_b = 2r_b+1$. And G is a network (Figure 2) containing a set of nodes $N = \{1, 2, ..., n\}$ and edges $E = \{1, 2, ..., m\}$, in which the distances between routers must be strictly less than l_b.

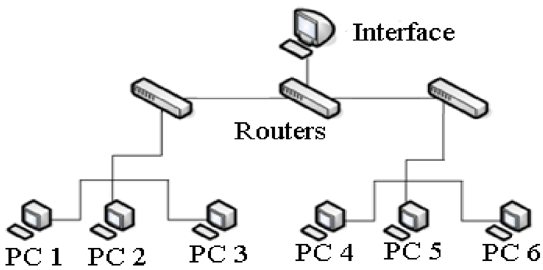


Fig. 2. Network topology example.

Next, we determine the eccentricity *e(v)* of the nodes v of the connected network *G* by the formula:

$$e(V) = max\{d(v,u|u \,\epsilon\, V(G)\} \tag{1}$$

The eccentricity *e(v)* of node *v* in the connected network *G* is the maximum distance between nodes *u* and *v*. Thus, the eccentricity of the network is the maximum distance between network nodes [28].

Further in the figure. 3 we present an implementation for dividing the network into clusters using network eccentricity. For example, the network includes 13 nodes and 12 edges, as shown in Figure 3.

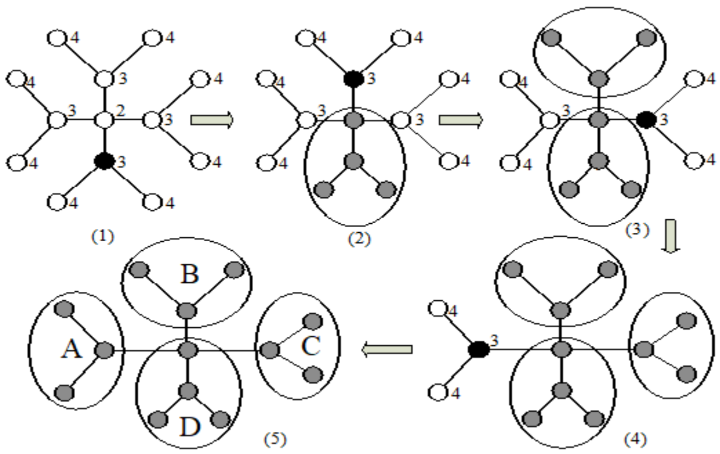


Fig. 3. Division of the network *G* into clusters (A, B, C, D) based on eccentricity (r_b=1). White nodes are uncovered nodes, gray nodes are covered, black nodes are centers. Numbers nodes indicate eccentricities.

At the first stage, we find the eccentricity *e(v)* of the nodes and the central node. As the central node, we choose the node located at a distance r_b from the node with the maximum eccentricity. At the second stage, nodes located at a distance r_b from the central node are covered by one cluster with the central node. In the next steps, we continue operations as in the previous steps until the entire network is covered. After the entire network is covered with clusters with radius r_b, the shortest path between clusters is first determined (Figure 4), and then between nodes (Figure 5).

After the entire network is covered with clusters with radius r_b, the shortest path between clusters is first determined (Figure 4), and then between nodes (Figure 5). The cluster routing process includes the following phases: first, we need to find the shortest path from node 1 to node 2 (Figure 5). To do this, the network is divided into four clusters (A, B, C, D) using the cluster routing algorithm, and the nodes in each cluster are covered with lines.

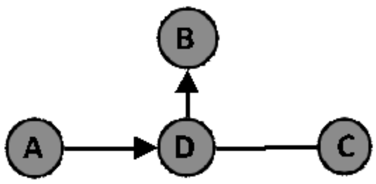


Fig. 4. The shortest path between clusters of the network from FIG.

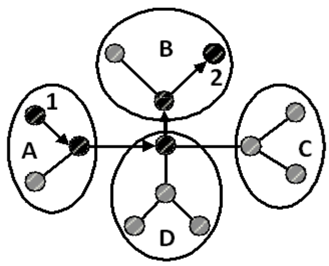


Fig. 5. Illustration of the global shortest path with r_b=1.

Second, we represent each cell as a single node. If there is an edge between two nodes in two different cells, we consider these two cells to be connected. The network shown in fig. 4 consists of four different blocks formed by the network shown in fig. 3. It is obvious that the size of the network has decreased significantly. Thirdly, we find the shortest path between nodes 1 and 2 (arrows) as shown in Fig. 5.

Thus, the number of operations performed to find the shortest path between nodes is reduced. All of the above operation for the compilation of cluster routing is carried out on the FPGA 5 (Fig. 1).

2. Entropy and the Renyi, Tsallis information dimensions of cluster router

To calculate the uncertainty of a probability distribution, Shannon first proposed the concept of Shannon's entropy. In modern communication theory the Shannon entropy has become one of the best known measures of uncertainty, and it is used mainly to describe uncertainty relations and predictions of quantum mechanics. The Shannon entropy [23] is determined by the following formula:

$$H(X) = \sum_k^n p_k \log_2 \frac{1}{p_k}, \quad (2)$$

where $X = (p_1, p_2, \quad p_k)$ is finite discrete probability distribution, which means $p_k \geq 0\ (k = 1,2 \dots \dots n)$ и $\sum_{k=1}^n p_k = 1$.

With the development of information theory, the Shannon entropy in some cases may not meet all the requirements for application in various fields. To further apply entropy and other options for calculating the uncertainty of a generalized distribution, Alfred Renyi introduced a new entropy inspired by Shannon's entropy. The Renyi entropy is a family of entropies that can be used in special cases by changing the parameter α. This entropy also can preserve additivity. The Renyi entropy [18] of order α is defined as:

$$(X) = \frac{1}{1-\alpha} \log \sum_{k=1}^n p_k{}^\alpha . \quad (3)$$

For $\alpha \in (0, 1) \cup (1, \infty)$ and the corresponding limit for $\alpha \in \{0, 1, \infty\}$. For $\alpha = 1$, the limit of the Renyi entropy can correspond to the Shannon entropy; therefore, the Renyi entropy is a generalization of the Shannon entropy. The Tsallis' entropy of order q can be defined as follows:

$$S_q = k \frac{1 - \sum_{i=1}^N p_i^q}{q-1}, \quad (4)$$

where N is the total number of elements in the set of probabilities, and p_i are the corresponding probabilities. When the order of q is 1, the Tsallis entropy corresponds to the Shannon entropy. The order of α in Renyi entropy is usually compared with the order q in the Tsallis' entropy to analyze their stability in rapidly and slowly changing situations [22].

As for the information dimension, it was first used to estimate the information load and measure strange attractors. For the first time, Wei et al. defined an information dimension [26] based on the information entropy and the box covering algorithm. The information contained in a complex network can be defined as follows:

$$I = -\sum_{i=1}^{N_B} p_i \ln p_i, \quad (5)$$

where p_i is the probability of nodes in any box, and can be defined as follows:

$$p_i = \frac{n_i}{n}, \quad (6)$$

where n_i is the number of nodes in any box, and n is the total number of nodes in any network. And the information dimension of the network can be calculated as follows:

$$d_I = -\lim_{l \to 0} \frac{I}{\log l_B} = \lim_{l \to 0} \frac{\sum_{i=1}^{N l_B} \frac{n_i(l_B)}{n} \log\left(\frac{n_i(l_B)}{n}\right)}{\log l_B}, \quad (7)$$

where l_B is the length of the box needed to cover the network, $\frac{n_i(l_B)}{n}$ is the probability of nodes in any box, and the edge of the box is equal to l_B [20].

The informational dimension of Tsallis was proposed by Zhang [23] to explain the complexity of the structure of networks and to reflect the degree of self-similarity and fractal properties. The Tsallis information dimension can be calculated as follows:

$$\boldsymbol{d}_{\mathrm{T}} = -\lim_{l_B \to 0} \frac{\frac{1-\sum_{i=1}^{N_B} p_i(l_B)^q}{q-1}}{\ln l_B}, \tag{8}$$

According to equation (7), the Tsallis entropy information can be found as follows:

$$I_{\mathrm{T}} = \frac{\sum_{i=1}^{N_B} p_i\,(l_B)^q - 1}{1-q}, \tag{9}$$

where $\boldsymbol{l_B}$ is the length covering the box and $\boldsymbol{p_i}$ is the probability associated with the box coverage results. q is the limiting parameter of the generalized entropy.

There are many dimensions to describe the complexity and uncertainty of a network, but many of them have fixed formulas and are not flexible. And the Renyi information dimension has a parameter α, which can change and affect the measurement itself. The Renyi dimension is defined as follows:

$$d_{\mathrm{R}} = -\lim_{l_B \to 0} \frac{\frac{1}{1-\alpha}\log \sum_{k=1}^{n} p_k{}^{\alpha}}{\log l_B}, \tag{10}$$

where $\boldsymbol{l_B}$ is the box size of the box covering algorithm. For α = 1, the Renyi dimension corresponds to the information dimension, which is easily proved using the L'Hospital equation. When α = 0, the dimension is exactly the classical Hausdorff dimension [26].

3. Calculation of the Renyi and Tsallis information dimensions of for the UV-flower model network

UV-flower is a model network that has a certain structure [24]. In the first generation (n=1) we start building a circular graph U+V, where U and V are network parameters. In the next generation (n=2) we replace each node with two parallel edges U and V. These operations are shown in Figure 6.

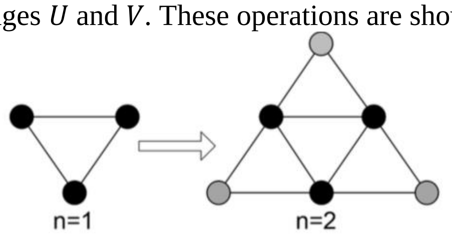


Fig. 6. The first two generations of the UV-flower model network.

In Table 1 below, we present the Renyi and Tsallis information dimensions of the UV-flower model network by dividing this network with different box covering algorithms.

Table 1. Reniy and Tsallis information dimensions of the UV-flower model network

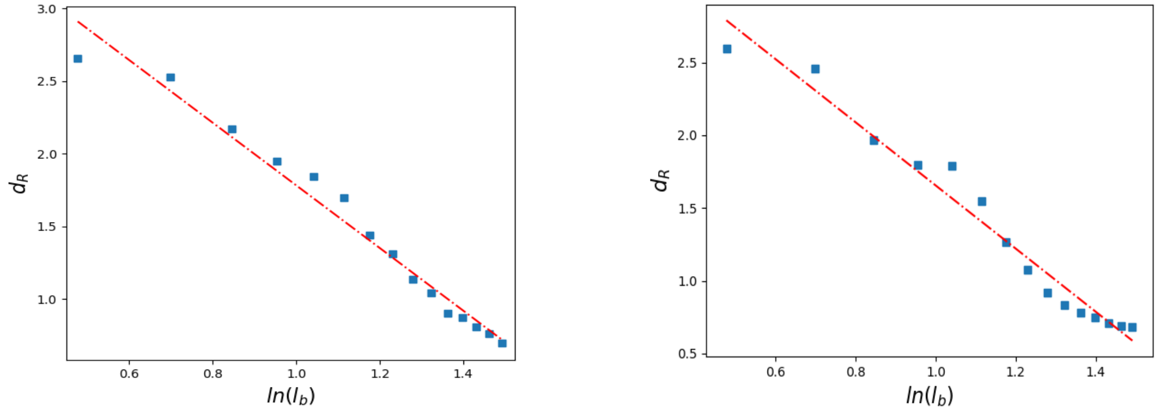
Box covering algorithms	Reniy information dimensions (α=0.5)	Reniy information dimensions (α=2)	Tsallis information dimensions (q =0.5)	Tsallis information dimensions (q =2)
MEMB	1.820	1.674	1.821	1.674
GC	1.823	1.783	1.820	1.798
RS	1.992	1.946	2.042	1.945
CIEA	2.160	2.170	2.158	2.169

The theoretical fractal dimension of the UV-flower network is determined by the following formula:

$$\boldsymbol{D} = \frac{\ln(\boldsymbol{U}+\boldsymbol{V})}{\ln \boldsymbol{U}}, \qquad \boldsymbol{U} > \mathbf{1} \tag{11}$$

According to formula 10, the theoretical fractal dimension [17] of the UV-flower model network is D = 2.321, which can be compared with the experimental data in Table 1 and reveal the most accurate result.

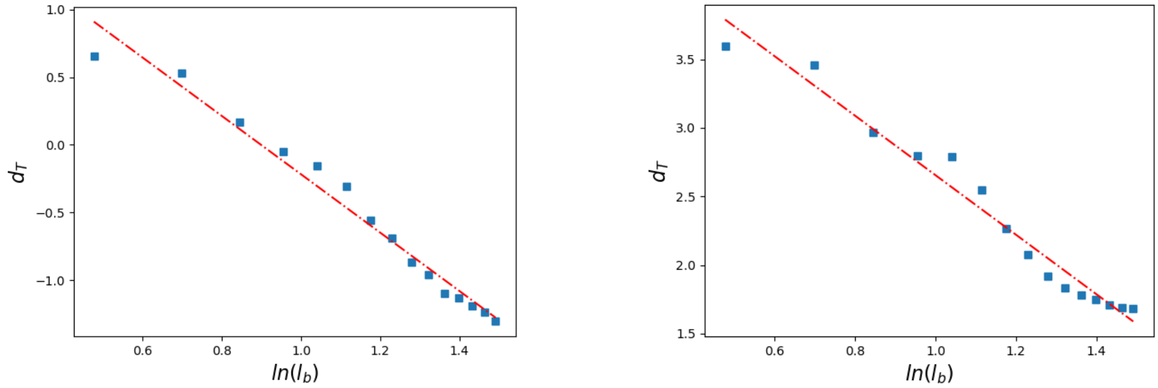
Figure 7 (a, b) below shows the Renyi information dimension for the UV-flower model network, partitioned using the CIEA.



a *b*

Fig. 7. Renyi dimension (d_R = 2.16) when separating the UV-flower network by the CIEA:
a) d_R = 2.16, α = 0.5; b) - d_R = 2.17, α = 2.

Figure 8 (a, b) below shows the Tsallis information dimension for the UV-flower model network, partitioned using the CIEA.



a *b*

Fig. 8. Tsallis dimension when separating the UV-flower network by CIEA:
a) d_T = 2.158, q = 0.5; b) d_R = 2.16, q = 2:

According to Figure 7, 8 the power rule for the cluster size is satisfied, which reflects the fractal property. Since we know that the theoretical dimension of the UV flower model network is 2.32, we get closer value when parameters q and α equal to 2 (d_T = 2.169 and d_R = 2.17) and slightly less when the parameters equal to 0.5.

Conclusion

In this paper, a cluster router was proposed that performs rapid route construction by clustering the network based on eccentricity, which ensures the autonomy of the device in case of failures. This router, unlike other routers based on the Dijkstra algorithm, builds a route first between clusters and then between network nodes, which allows you to significantly succeed in speed. To analyze this router we calculated the Tsallis and Renyi information dimensions of the UV flower model network using box covering algorithms. The values closest to the theoretical values were obtained when the network was covered by the CIEA and the information dimension of Tsallis was D_T = 2.158 (for q = 0.5) and D_T = 2.169 (for q = 2), and the Renyi dimension D_R = 2.16 (for α = 0.5) and D_R = 2.17 (for α = 2).

Acknowledgement

We would like to express our sincerest gratitude to the Research Institute of Experimental and Theoretical Physics of the Al-Farabi Kazakh National University for supporting this work by providing computing resources (Department of Physics and Technology). This research was funded by the Committee of Science of the Ministry of Education and Science of the Republic of Kazakhstan, grant AP09058525.

REFERENCES

1 Mazloomi N., Gholipour M., Zaretalab A., Efficient configuration for multi-objective QoS optimization in wireless sensor network. Ad Hoc Networks. 2022, 125 p.

2 Mehbodniya A., Bhatia S., Mashat A., et al. Proportional fairness based energy efficient routing in wireless sensor network. *Computer Systems Science and Engineering*. 2022. Vol. 41(3), pp. 1071-1082.

3 Nasri M., et al. Energy-efficient fuzzy logic-based cross-layer hierarchical routing protocol for wireless internet-of-things sensor networks. *International Journal of Communication Systems*. 2021.Vol.34(9), pp. 4808.

4 Khan M.K., et. al., Hierarchical routing protocols for wireless sensor networks: functional and performance analysis. *Journal of Sensors*. 2021. pp. 1-18.

5 Al-Zubi R. T., et al. Solution for intra/inter-cluster event-reporting problem in cluster-based protocols for wireless sensor networks. International *Journal of Electrical and Computer Engineering*. 2022. Vol. 12(1), pp. 868-879.

6 Daanoune I., Abdennaceur B., Ballouk A. A comprehensive survey on LEACH-based clustering routing protocols in wireless sensor networks. *Ad Hoc Networks*, 2021, Vol.114, pp.102409.

7 Blazevic L., Le Boudec J.Y., Giordano S. A location based routing method for mobile ad hoc networks. *IEEE Transactions on Mobile Computing*. 2005. Vol.4, pp. 97–110.

8 Sarkar A., Murugan T.S., Cluster head selection for energy efficient and delay-less routing in wireless sensor network. *Journal of Wireless Networks*. 2019. Vol. 25, No. 1, pp. 303– 320.

9 Soni V., Mallick D.K. Location-based routing protocols in wireless sensor networks: A survey. *International Journal of Internet Protocol Technology*. 2014. Vol. 8(4), pp. 200 – 213.

10 Huang R., Cui X., Pu H. Wireless sensor network clustering routing protocol based on energy and distance. *Lecture Notes in Electrical Engineering*. 2020. Vol. 675, pp. 1735 – 1745.

11 Song C., et al. How to calculate the fractal dimension of a complex network: the box covering algorithm. *Journal of Statistical Mechanics: Theory and Experiment*. 2007. Vol. 2007, No.03, pp. P03006.

12 Schneider C.M., et al. Box-covering algorithm for fractal dimension of complex networks. *Physical Review E.* 2012. Vol. 86, No.1, pp. 016707.

13 Song C., Havlin S., Makse H.A. Origins of fractality in the growth of complex networks. *Nature physics*. 2006. Vol.2, No.4, pp. 275 – 281.

14 Gallos L.K., Song C., Makse H.A. A review of fractality and self-similarity in complex networks. *Physica A: Statistical Mechanics and its Applications.* 2007. Vol. 386, No.2, pp. 686 – 691.

15 Wen T., Cheong K. H. The fractal dimension of complex networks: A review. *Information Fusion*. 2021. Vol. 73, pp.87 – 102.

16 Akhtanov S. et al. Centre including eccentricity algorithm for complex networks. *Electronics Letters*. 2022. Vol. 58, No. 7, pp. 283 – 285.

17 Kim J.S., et al. Fractality in complex networks: Critical and supercritical skeletons. *Physical Review E.* 2007, Vol.75. No.1, pp. 016110.

18 Long G.U.O., Cai X.U. The fractal dimensions of complex networks. *Chinese Physics Letters*. 2009, Vol.26. No.8, pp. 088901.

19 Zhang Q., et al. Tsallis information dimension of complex networks. *Physica A: Statistical Mechanics and its Applications*. 2015. Vol. 419, pp. 707 – 717.

20 Ramirez-Arellano A., Hernandez-Simon L.M., Bory-Reyes J. A box-covering Tsallis information dimension and non-extensive property of complex networks*. Chaos, Solitons & Fractals*. 2020. Vol.132, pp. 109590.

21 Bromiley P. A., Thacker N. A., Bouhova-Thacker E. Shannon entropy, Renyi entropy, and information. *Statistics and Inf. Series* (2004-004). 2004. Vol. 9, pp. 2 – 8.

22 Zyczkowski K. Renyi extrapolation of Shannon entropy. *Open Systems & Information Dynamics*. 2003. Vol.10, No.03, pp. 297 – 310.

23 Wei D., et al. A new information dimension of complex networks. *Physics Letters A.* 2014. Vol.378, No.16-17, pp.1091 – 1094.

24 Farmer J.D. Information dimension and the probabilistic structure of chaos. *Zeitschrift für Naturforschung A.* 1982. Vol. 37, No.11, pp. 1304 – 1326.

25 Geiger B.C., Koch T. On the information dimension of stochastic processes. *IEEE transactions on information theory*. 2019. Vol.65, No. 10, pp. 6496 – 6518.

26 Barth A., Baumann G., Nonnenmacher T. F. Measuring Renyi dimensions by a modified box algorithm. *Journal of Physics A: Mathematical and General*. 1992. Vol. 25, No.2, pp. 381.

27 Zhang Q. et al. Tsallis information dimension of complex networks. *Physica A: Statistical Mechanics and its Applications*. 2015. Vol. 419, pp. 707 – 717.

28 Duan S., Wen T., Jiang W. A new information dimension of complex network based on Rényi entropy. *Physica A: Statistical Mechanics and its Applications*. 2019. Vol.516, pp. 529 – 542.

Article accepted for publication 28.09.2022