DEVELOPMENT OF ENERGY ANALYZER OF CHARGED PARTICLES BASED ON THE BASIS NON-UNIFORM ELECTROSTATIC FIELD

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The electron-optical characteristics of a mirror energy analyzer based on a non-uniform, hyperbolic decreasing electrostatic field were investigated. In work an approximate-analytical method for calculating the trajectory of charged particles in an electrostatic field, acting in the mirror reflection regime, was used. The equation of motion of charged particles in the integro-differential form is obtained. The scheme of the most optimal variant of the analyzer with a non-uniform field is found.

Keywords: non-uniform field, energy analyzer, mirror reflection regime, electron-optical characteristics, trajectory of charged particles.

Introduction

The great potential of electron spectroscopy, associated with the broad information that the energy spectrum of these particles carries about various physical processes, about the matter structure, stimulates further improvement of the known analysis methods and the development of new directions of electron spectroscopy.

At the initial stage of the development of electron optics, the main research focused on the study of axially symmetric fields with the stigmatic focusing properties of a charged particles beam and the formation of a correct scale-free undistorted image. Electrostatic mirror energy analyzers based on cylindrical, spherical and hyperbolic fields turned out to be the most advanced and widely used devices in the study of small and medium energy electron beams. Among the known classical type fields an electrostatic mirror with a uniform field is the simplest in construction and widely used energy analyzer of charged particle beams.

Further progress in the development of effective methods for studying the solid surface requires a significant modernization of existing or the creation of qualitatively new analyzing systems based on the further development of the theory. The development of high-resolution electron analyzers based on the synthesis of multipoles and a cylindrical field can be attributed to qualitatively new methods [1-10]. To confirm the universality of the obtained results, it is necessary to continue research of analytical systems based on the synthesis of multipoles with other types of classical electrostatic fields. This class of potential fields includes an non-uniform electrostatic field decreasing by a hyperbolic law, which is a superposition of a dipole with a uniform field.

1. Calculation and analysis of the electron-optical properties of an electrostatic non-uniform field

The object of the study is electrostatic non-uniform field acting in the mirror reflection regime, that decreasing by hyperbolic law. The potential is described by the expression

$$U = \frac{U_0}{d} y(1 - Az) ,$$ (1)

where A is dimensionless parameter, at $A=0$ - field (1) is uniform.

The profile of the outer deflecting electrode is determined by calculation the equipotential lines in a non-uniform field. Fig.1 shows a portrait of equipotential lines in an electrostatic non-uniform field at $A = 0.01$.
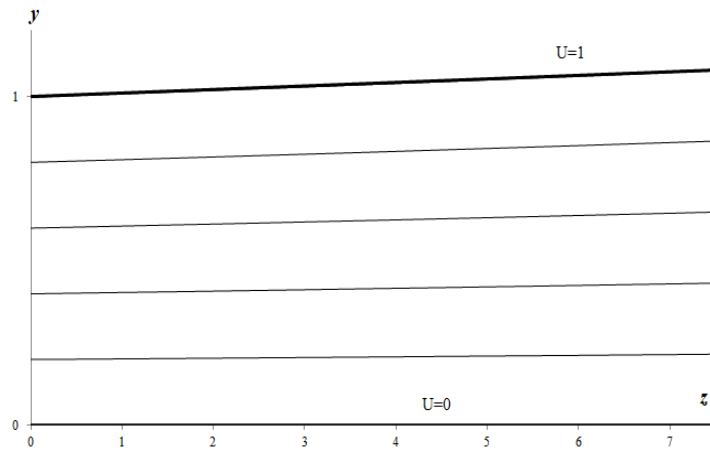
Fig.1. Equipotential lines in a non-uniform field at $A = 0.01$

A field (1) is formed in the space between two electrodes, one of which remains flat and is under the ground potential, the deflecting potential U_0 is supplied to the other electrode, which has a hyperbolic profile (Fig.2).
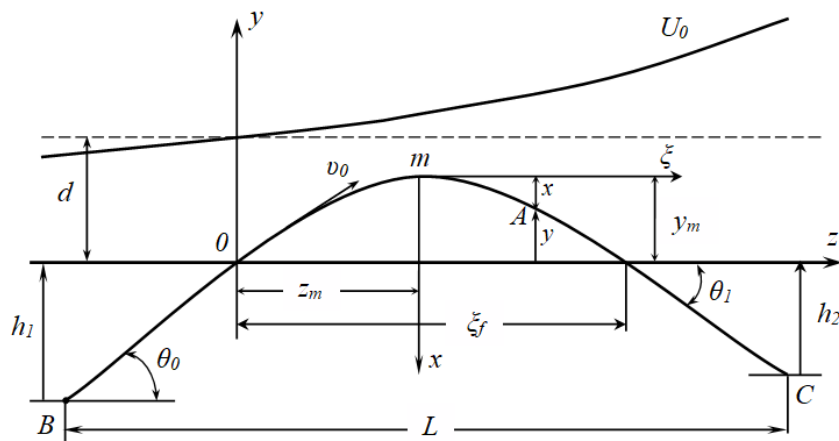
Fig.2. The electrostatic mirror based on non-uniform field.
The dashed line is a flat electrode in the limiting case of a flat mirror
(B is source of charged particles; C is electron-optical image of the source)

In the mirror reflection regime a charged particles beam enters field (1) at an angle θ_0 to the z axis, moves along a "return" trajectory having a vertex **m** in the electrostatic field region, and returns to the lower electrode at an angle θ_1. The return trajectory of charged particles consists of right and left branches, asymmetrical about the trajectory vertex **m**, therefore, a separate calculation of each of its branches and their subsequent joining is necessary. The condition for joining the right

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and left branches of the trajectory is the equality of the functions describing the particles trajectory and their derivatives at the point **m** for both branches. For calculate the branches of the return trajectory, let's move to a new coordinate system, the beginning of which is located at the trajectory vertex **m**.

According to Fig.2,

$$z = z_m \pm \xi, \quad x = y_m - y. \tag{2}$$

Here, the upper sign in front ξ corresponds to the right branch ($\xi > 0$), the lower sign to the left branch of the trajectory ($\xi < 0$). The distribution of the field (1) in the coordinate system x, ξ has the following form:

$$U(x,\xi) = \omega(1 \pm A'\xi)(y_m - x), \tag{3}$$

where

$$\omega = \frac{U_0(1 - Az_m)}{d}, \qquad A' = \frac{A}{1 - Az_m}. \tag{4}$$

According to the law of conservation of energy when moving in an electrostatic potential field, the kinetic energy of a charged particle is determined by the passed potential difference. For a particle moving in the field (3) from point **m** to an arbitrary point A, we can write:

$$\frac{m}{2}(\dot{x}^2 + \dot{\xi}^2) - \frac{m\upsilon_m^2}{2} = -q(U_m - U) = q\omega G_{1,2}(x, \xi), \tag{5}$$

where

$$G_{1,2}(x,\xi) = x \pm A'\xi(y_m - x). \tag{6}$$

Here and below, the number 1 in the subscript corresponds to the functions for the right branch, the number 2 to the functions of the left branch of the trajectory.

The law of conservation of energy for the longitudinal component of the motion of a charged particle in the field (3), taking into account the condition $\upsilon_m^2 = \dot{\xi}_m^2$, because $\dot{x}_m = 0$, as well as the ratio $\frac{d\xi}{dt} = \frac{d\xi}{dx}\frac{dx}{dt} = \xi'\dot{x}$, will be written as follows

$$\frac{m\dot{\xi}^2}{2} - \frac{m\upsilon_m^2}{2} = -q\int_0^x \frac{\partial U(x,\xi)}{\partial \xi}\xi'd\xi = \mp q\omega A'\int_0^x (y_m - x)\xi'd\xi, \tag{7}$$

where the derivative ξ' is greater than zero for both branches, as x and ξ are taken in absolute value.

At $x = y_m$, $\frac{m\dot{\xi}^2}{2} = \frac{m\upsilon_0^2}{2}\cos^2\theta_{0,1} = W\cos^2\theta_{0,1}$, therefore, expression (7) can be rewritten with respect to $\frac{m\upsilon_m^2}{2}$ the following form:

$$\frac{m\upsilon_m^2}{2} = q\omega\left(S'\cos^2\theta_{0,1} \pm A'f_{m_{1,2}}\right), \tag{8}$$

where

$$S' = \frac{S}{1 - A z_m}, \qquad S = \frac{Wd}{qU_0}, \qquad f_{m_{1,2}} = \int_0^{y_m} (y_m - x)\xi' dx \qquad (9)$$

The value of S has the dimension of length and is some characteristic size of the electron mirror. Solving equations (5), (7) and (8) relatively to ξ', we come to the integro-differential equation of the trajectory of a charged particle in a non-uniform field (3)

$$\left(\xi'_{1,2}\right)^2 \left[G_{1,2}(x,\xi) \pm A' \int_0^x (y_m - x)\xi'_{1,2}\, dx \right] =$$
$$= S' \cos^2 \theta_{0,1} \pm A' f_{m_{1,2}} \mp A' \int_0^x (y_m - x)\xi'_{1,2}\, dx. \qquad (10)$$

The integral-differential equation (10) has a singular point at $x=0$, since the denominator in this case vanishes, so the solution of the equation is sought as a generalized power series [11]:

$$\xi = \sqrt{x} \sum_{n=0}^{\infty} c_n x^n + \sum_{n=1}^{\infty} a_n x^n. \qquad (11)$$

Final results of the calculation of the total projection of the particle trajectory onto the z axis from point source B to its image C are presented below. The equations are obtained in units of the parameter S, which has the dimension of length. According to Fig.2, the total projection of the trajectory from the source to its image is the sum:

$$l = \frac{L}{S} = \frac{1}{S}\left(H_1 \tan\theta_o + H_2 \tan\theta_1 + \xi_f \right) = \left(h_1 \tan\theta_o + h_2 \tan\theta_1 + \xi_f \right) \qquad (12)$$

where in

$$\frac{\xi_f}{S} = 2\sin 2\theta_o + SA\left(\frac{4}{3}\sin^2 2\theta_o + \frac{8}{3}\sin^4 \theta_o \right) + (SA)^2 \left(\frac{16}{9}\sin^3 2\theta_o + \frac{16}{3}\sin^4 \theta_o \sin 2\theta_o \right), \qquad (13)$$

and the inclination angle of the trajectory to the axis at the exit point of the trajectory from the field:

$$\tan\theta_1 = \tan\theta_0 + \frac{4}{3}(SA) + \frac{8}{3}(SA)^2 \sin 2\theta_0. \qquad (14)$$

Considering the divergence angle $\Delta\theta$ of the beam in the axial plane and the relatively small value of energy spread $\varepsilon = \frac{\Delta W}{W}$ in the beam as small perturbations, one can decompose L into a Taylor series [12]:

$$L = L_o + \frac{\partial L}{\partial\theta}\Delta\theta + \frac{\partial L}{\partial\varepsilon}\Delta\varepsilon + \frac{1}{2!}\left(\frac{\partial^2 L}{\partial\theta^2}(\Delta\theta)^2 + \frac{\partial^2 L}{\partial\varepsilon^2}(\Delta\varepsilon)^2 + \frac{\partial^2 L}{\partial\theta\,\partial\varepsilon}\Delta\theta\,\Delta\varepsilon \right) + \dots \qquad (15)$$

The second-order angular focusing regime is determined from the condition $\dfrac{dL}{d\theta} = \dfrac{d^2 L}{d\theta^2} = 0$

$$\cos^2 \theta_o - 3\sin^2 \theta_o + (SA)\sin 4\theta_o +$$

$$+\frac{4}{3}(SA)^2 \left[\frac{H_2}{S}\left(\cos^2 \theta_0 - 3\sin^2 \theta_o\right) + \sin^2 \theta_o \left(16\cos^2 \theta_0 - 10\sin^2 \theta_o\right) \right] = 0 . \qquad (16)$$

From the analysis of equation (16) it has been established that second-order angular focusing regime can be realized only for the analyzer scheme with parameters $A=0$, $h_1 + h_2 = 0,5$, i.e. for a flat mirror. To search for the parameters of the most optimal variant of analyzer with a non-uniform field decreasing by hyperbolic law, it is necessary to determine the functions $f(\Delta\theta, SA, h_1/h_2) = l(\theta) - l(\theta_o)$ characterizing the longitudinal aberration smearing of the image near the Gaussian focus by the formula (12) for different values of SA and the selected divergence angle $\Delta\theta$ of the analyzed beam.

The aberration smearing functions of electron-optical mirrors with a non-uniform field (Table 1), whose schemes correspond to different values of SA, were calculated and tuned to the second-order angle focusing regime of the flat mirror $h_1 + h_2 = 0,5$, $\theta_0 = 30^0$.

Table 1. The aberration smearing functions of electron mirrors with a non-uniform field

SA	θ_1 (deg)	l_θ	$f(\Delta\theta, SA, h_1/h_2)$
0.000	30	2.5981	0.0268
0.005	29.9043	2.6056	0.0154
0.010	29.8082	2.6133	0.0117
0.015	29.7118	2.6210	0.0079
0.020	29.6149	2.6289	0.0063
0.025	29.5178	2.6368	0.0074
0.030	29.4202	2.6449	0.0097
0.035	29.3224	2.6530	0.0125
0.040	29.2242	2.6613	0.0157
0.045	29.1257	2.6697	0.0188
0.050	29.0269	2.6781	0.0220

From the results of calculation the trajectories performed for the angular spread $\Delta\theta = \pm 6°$ of particles at the analyzer entrance, a scheme was found for the most optimal variant of analyzer with a non-uniform field: $h_1 + h_2 = 0,5$, $\theta_0 = 30^0$ and $SA = 0.02$. In this case the value of the aberration smearing $f(\Delta\theta, SA, h_1/h_2)$ is 3 times less than the flat mirror ($SA = 0$). This means that in a mirror analyzer built on the basis of a non-uniform electrostatic field that decreasing by a hyperbolic law, which is a superposition of a dipole with a uniform field, the resolution can be improved several times as compared with the case of a flat mirror.

Conclusion

A theoretical study of the electron-optical properties of an analyzing system based on a mirror with a modified electrostatic field has been carried out. Equation of total projection of the particle trajectory on the axis from the source to the image was obtained. The aberration smearing functions of electron mirrors with a non-uniform field were calculated. The optimal variant of the analyzer scheme with a non-uniform field has a higher resolution than a flat mirror.

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