

## APPLICATION OF A COMPLETE MULTI-NETWORK METHOD FOR SOLVING THE PROBLEM OF FLOWS AROUND SPHERE

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*Using the example of a numerical solution of the classical problem of a viscous flow past a sphere, the efficiency of using the multigrid method was compared with direct calculations. Various difference schemes are considered. It's shown that to go from grid to grid, for a vortex equation, it is necessary to use a 9-point pattern. For different Reynolds numbers the resistance coefficient to friction had been calculated. The limit on the Reynolds number when using the multigrid method is noted.*

**Keywords:** numerical solution, multigrid method, viscosity, flow, sphere, ellipsoid.

### Introduction

The increasing complexity of the considered natural and technological problems currently facing humanity increases the requirements for methods of solving such problems. One of the common ways to obtain information is a numerical experiment. Therefore, the development of efficient algorithms was, is and will be a relevant research goal. The development of technology has led to the possibility of using multiprocessor devices and, as a result, to the possibility of using more efficient computing technologies. One of which is the multigrid method that allows parallelizing the numerical algorithm.

As is known, the essence of the full multigrid method (FMM) is based on the sequence of nested grids. Cyclic transitions from one grid to another allow one to effectively suppress the low-frequency components of the residual of the solution when calculating with relaxation difference schemes [1]. The ideology of the method allows the use of parallel computing on multiprocessor technology.

This paper presents the results of a study of the effectiveness of the multigrid method for determining the coefficient of friction resistance with an external stationary flow of an incompressible viscous fluid around a solid sphere. This problem is a classical problem of hydrodynamics, which has solutions obtained by analytical and numerical methods. Therefore, it can serve as a test for approbation of the considered solution method.

The relevance of the chosen class of tasks is undoubted. The movement of multiphase media in a variety of technological and natural processes is of great interest. The solution of the complete problem taking into account the interaction of each particle with the carrier medium is an insoluble problem due to the large number of moving particles. As a rule, they are limited by the phenomenological law of the hydrodynamic resistance of a single spherical or spheroidal particle in a carrier medium. The assumption of spherical particle shape is the most common and quite acceptable.

This approach reasonably led to the fact that most of the efforts to determine the law of resistance was devoted to the study of spherical particles. There are many such works, it is enough to specify [2, 3]. In many cases, a moving particle can more accurately be described as an ellipsoid. And in this regard, of interest are studies devoted to the flow around ellipsoids, in particular, ellipsoids of rotation (spheroids).

In [4-5], the results of analytical calculations for the flow parallel to the main axis of spheroids at small Reynolds numbers are presented. In [6-7], the problem of slow viscous flow of a stationary flow around a triaxial ellipsoid was solved analytically based on the application of a tension-compression transformation, and a simple calculation formula for its resistance was indicated. In [8], this method allowed the solution of a non-stationary problem of determining the resistance force during a slow rotation of an ellipsoid in a viscous fluid.

From the examples of numerical studies, it can be noted [9] where the results of the calculation of the flow around a uniform stationary flow of spheroids of various shapes with the numbers  $Re < 100$  in the axisymmetric formulation are presented. The calculated data on the flow around spheroids at Reynolds numbers around 100 are shown [10–13]. The results of numerical and experimental studies allow us to construct correlation dependences used in determining the resistance forces during the motion of particles.

## 1. Statement of the problem

The peculiarity of the considered problem, which is solved in variables vorticity - current function is the need to solve a system of two interrelated equations. We used the sequential method of solving this system:

- 1) the current function is determined for a given distribution of the vortex function;
- 2) the definition of the vortex function.

The process is repeated iteratively until it is established. Thus, it is necessary to perform a large number of repeated calculations and there is a good reason to assume a significant acceleration of the calculation process when using the multigrid method.

The system of equations describing the stationary flow of a viscous incompressible fluid around a sphere in terms of vorticity – the current function in spherical coordinates is as follows:

$$E^2 \psi = \zeta r \sin \theta$$

$$\frac{R}{2} \left[ \frac{\partial \psi}{\partial r} \frac{\partial}{\partial \theta} \left( \frac{\zeta}{r \sin \theta} \right) - \frac{\partial \psi}{\partial \theta} \frac{\partial}{\partial r} \left( \frac{\zeta}{r \sin \theta} \right) \right] \sin \theta = E^2 (\zeta r \sin \theta), \quad (1)$$

$$\text{where the operator } E^2 = \frac{\partial^2}{\partial r^2} + \frac{\sin \theta}{r^2} \frac{\partial}{\partial \theta} \left( \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \right).$$

All values are dimensionless as follows:

$$r = \frac{r'}{A}, \quad \psi = \frac{\psi'}{UA^2}, \quad \zeta = \frac{\zeta' A}{U}, \quad R = \frac{2UA}{\nu}. \quad (2)$$

Here  $A$  is the radius of the sphere,  $U$  is the velocity of the undisturbed flow, and  $\nu$  is the kinematic viscosity.

The boundary conditions are determined from the assumptions about the absence of perturbations at the remote boundary, the no-slip conditions on the solid sphere, and the axisymmetry of the flow. As the outer boundary, a surface is chosen that is seven radii from the surface of the sphere.

## 2. Method of solution

In the calculations, difference schemes were used - an explicit Gauss-Seidel scheme, an implicit longitudinal sweep scheme for a variable  $r$ . And these schemes were used in the multigrid method. To go from grid to grid, for a vortex equation, it is necessary to use a 9-point pattern, which allows to suppress nonphysical oscillations inherent to difference schemes with increasing Reynolds number [1]. The main characteristic for confirming the reliability of the results obtained

is the coefficient of resistance to friction, which was calculated for different numbers of Re: 0.01, 0.1, 0.5, 2, 5, 10, 20 (Table 1).

The numerical solution of the problem was carried out on a sequence of nested grids 25x25, 50x50, 100x100, 200x200.

### 3. Results and discussion

The results of numerical calculations were compared with the results of Jensen's research [1] for Reynolds numbers 5, 10, and 20 and with the well-known analytic Stokes dependence for the drag coefficient for friction in the flow at small Reynolds numbers. The data are shown in table 1.

**Table 1.** Friction drag coefficient

Re	Stoks 16/Re	Jenson	Multigrid explicit scheme	Explicit scheme	Multigrid implicit scheme	Implicit scheme
0.01	1600		1628.3	1683.5	1681	1630.6
0.1	160		163.09	168.23	169.16	163.37
0.5	32		33.77	34.73	33.57	33.94
1	16		17.50	17.63	17.48	17.60
2			10.30	10.54	9.72	10.29
5		5.34			4.98	5.42
10		3.10				3.37
20		1.857			1.94	1.97

Table 2 shows the comparative efficiency data of the multigrid method using different schemes for the direct calculation of the considered scheme. The relations of the time of the “usual” directly calculation on the smallest grid and the calculation time by the multigrid method using the same difference scheme and the same criterion for the termination of iterations in accuracy are given.

**Table 2.** Comparative effectiveness of the multigrid method.

Re	Multigrid explicit scheme	Multigrid implicit scheme
0.01	1.4	16.3
0.1	1.5	8.3
0.5	1.6	5.7
1	2.2	5
2	2.9	4
5		7.3

As can be seen from the table 2, the combination of the multigrid method and the implicit difference scheme seems to be the most effective. However, it is necessary to recognize that reducing the calculation time by 16 times seems “too” a good result.

The multigrid method has a peculiarity - the calculation on a coarse grid is limited by the stability criterion with respect to the grid Reynolds number. According to the recommendation of

Jenson [1], the grid step cannot exceed the value  $\frac{4}{Re}$ . Therefore, on the above sequence of grids, it is possible to obtain a solution only for Reynolds numbers less than 5. To calculate the flow field for Reynolds numbers 10 and 20, we used a sequence of grids with dimensions 50x50, 100x100, 200x200.

## Conclusion

The high efficiency of the application of the full multigrid method was confirmed by the example of the classical problem of viscous flow around a sphere. The reliability of the calculations is confirmed by comparison with the analytical and numerical results of other authors. The results show that the use of the full multigrid method allows to reduce the time for calculating tasks of this class on average by 4-7 times. There is a limitation on the size of the grid step, affecting the dimension of the nested grids used. These restrictions when applying this procedure on multiprocessor clusters require additional consideration.

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