

THE CALCULATION OF THE HEAT TRANSFER COEFFICIENT FROM THE WALLS AND PIPES OF THERMAL NETWORKS IN THE ENVIRONMENT

Sabdenov K.O., Sakipov K.Ye.

L.N. Gumilyov Eurasian National University, Nur-Sultan, Kazakhstan, sakamer2100@gmail.com

A simple semi-analytical method for determining the heat transfer coefficient from cylindrical surfaces is proposed. The idea of the method is based on the representation of the heat wave and the self-similar nature of the process, which allows to obtain an analytical formula. The physical explanation of heat transfer coefficient decrease is given. A comparison of the approximate solution with the numerical solution of the heat propagation problem shows the high accuracy of the analytical formula. The distribution of heat from the pipe into the cold surrounding space at large times with high accuracy can be considered a self-similar process.

Keywords: heat flow, pipeline, heat transfer coefficient, heat wave, self-similarity.

Introduction

When carrying out engineering calculations [1, 2] of heat losses by heat pipes in the environment, the heat transfer coefficient α in Newton's heat transfer law is an uncertain value

$$j = \alpha(T - T_c),$$

where j – the flow of heat; T_w – the surface temperature of the tube wall; T_c – temperature environment.

However, the heat transfer coefficient is assumed to be constant [1], this method can be taken only as an approximation, it gives acceptable results in a limited time interval. But a detailed analysis of the common methods for determining the heat flow j needs a thorough check and justification, since the problem of the distribution of heat from the heated pipe into the surrounding space is not stationary. This means the time variation of the heat transfer coefficient.

In this paper we propose a method for determining the heat transfer coefficient $\alpha(t)$ as a function of time t . For this purpose, the known [3] representation of the heat wave and the assumption of the approximate nature of the self-similar heat propagation are used. Below, a brief explanation of this representation is given by the example of heat propagation from an unbounded flat wall, after which the heat transfer by an infinite length pipe to the surrounding cold environment is analyzed.

1. Heat transfer coefficient between the wall with a constant high temperature and the environment.

Let there be a semi-bounded space (wall) with constant temperature T_w , the wall contacts a semi-bounded medium with low temperature T_0 : $T_w > T_0$. The wall occupies a half-space $x < 0$, the medium is located in the region $x > 0$. The equation of heat propagation in the environment with thermal diffusion coefficient κ is described by the equation

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}. \quad (1)$$

We will look for the temperature distribution in the form

$$T = T(z), \quad z = \frac{x^2}{4\kappa(t + t_*)},$$

where t_* – is the beginning of the self-similar heat propagation regime, this time must be found from the condition of equality of the heat flow from the exact solution of equation (1) and the heat flow found from the approximate solution of the same equation below.

Calculate the derivatives:

$$\frac{\partial T}{\partial t} = \frac{dT}{dz} \frac{\partial z}{\partial t} = -\frac{z}{t} \frac{dT}{dz}, \quad \frac{\partial T}{\partial x} = \frac{dT}{dz} \frac{\partial z}{\partial x} = \frac{x}{2\kappa t} \frac{dT}{dz},$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\kappa} \left(\frac{1}{2} \frac{dT}{dz} + z \frac{d^2 T}{dz^2} \right).$$

Substituting them into equation (1), we obtain

$$z \frac{d^2 T}{dz^2} + \left(\frac{1}{2} + z \right) \frac{dT}{dz} = 0.$$

Its General solution is

$$T = C_2 + C_1 \int_{z_0}^z \frac{e^{-\zeta}}{\sqrt{\zeta}} d\zeta,$$

where C_1, C_2 - are constant integrations.

Let $z = z_0$ be equal to $T = T_w$, then $C_2 = T_w$. Therefore

$$T = T_w + C_1 \int_{z_0}^z \frac{e^{-\zeta}}{\sqrt{\zeta}} d\zeta,$$

The wall and the environment begin to exchange heat at the initial time $t = 0$, after which the heated heat layer $x_0(t)$, which is a moving wave, "breaks away" from the wall (Fig. 1). To the left of this wave, the temperature $T \approx T_w$, is stored, to the right of the wave front, the temperature is a variable and depends on the x coordinate and time t . Therefore, the requirement used above

$$z = z_0: \quad T \approx T_w,$$

means statement of a boundary condition on the front of this heat wave.

The second boundary condition requires the equality $T = T_0$ for $z \rightarrow \infty$. Fulfilling this condition, we obtain:

$$T_0 = T_w + C_1 \int_{z_0}^{\infty} \frac{e^{-\zeta}}{\sqrt{\zeta}} d\zeta.$$

Denoted

$$\chi = \int_{z_0}^{\infty} \frac{e^{-\zeta}}{\sqrt{\zeta}} d\zeta,$$

Dene the constant C_1 :

$$C_1 = -\frac{T_w - T_0}{\chi}.$$

With this result, you can write

$$T = T_w - \frac{T_w - T_0}{\chi} \int_{z_0}^z \frac{e^{-\zeta}}{\sqrt{\zeta}} d\zeta. \quad (2)$$

The coordinate of the heated layer depends on the time as

$$x_0(t) = \sqrt{4z_0\kappa(t + t_*)}.$$

This shows the existence of arbitrariness in the choice of the constant z_0 . This arbitrariness arose due to the lack of a clear indication of what should be taken as the coordinate of the heat wave.

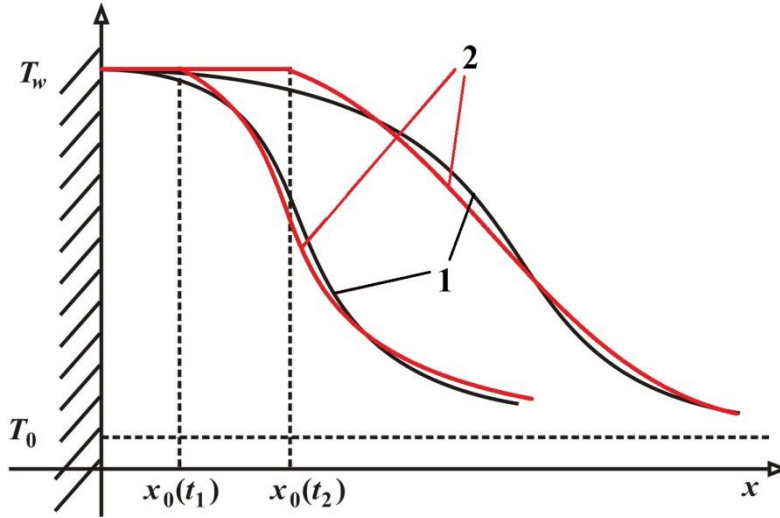


Fig.1. Location of the boundary $x_0(t)$ of the heat wave at two different times t_1 and t_2 ; 1 – real temperature profile; 2 – approximate profile.

But the integral in (2) converges at any (positive) values of z_0 , including $z_0 = 0$. Then it would be logical to choose the lower limit of integration equal to this zero value, and instead of (2) we obtain

$$T = T_w - \frac{T_w - T_0}{\chi} \int_0^z \frac{e^{-\zeta}}{\sqrt{\zeta}} d\zeta, \quad (3)$$

$$\chi = \int_0^\infty \frac{e^{-\zeta}}{\sqrt{\zeta}} d\zeta.$$

Find the heat flow j_w at the boundary of the systems, including on the one hand the wall with the heat wave and the environment on the other hand. This boundary is located at the point $x = x_0(t) = 0$, i.e. the "separation" of the heat wave does not occur. By definition

$$\begin{aligned} j_w &= -\lambda \left. \frac{\partial T}{\partial x} \right|_{x=x_0(t)} = -\lambda \left. \frac{dT}{dz} \right|_{z=z_0} \left. \frac{\partial z}{\partial x} \right|_{x=x_0(t)} = \lambda \frac{T_w - T_0}{\chi} \frac{e^{-z_0}}{\sqrt{z_0}} \frac{x_0(t)}{2\kappa(t + t_*)} = \\ &= \lambda \frac{T_w - T_0}{\chi} \exp(-z_0) \frac{2\sqrt{\kappa(t + t_*)}}{2\kappa(t + t_*)} = \frac{\lambda}{\sqrt{\kappa(t + t_*)}} \frac{e^{-z_0}}{\chi} (T_w - T_0). \end{aligned}$$

As a result, taking into account the equality $z_0 = 0$, the expression is obtained

$$j_w = \frac{T_w - T_0}{\chi} \frac{\lambda}{\sqrt{\kappa(t + t_*)}}.$$

This shows that the heat transfer coefficient α equal

$$\alpha = \frac{1}{\chi} \frac{\lambda}{\sqrt{\kappa(t + t_*)}}.$$

The decrease in the heat transfer coefficient is easy to explain from a physical point of view: as the cold space warms up, it will heat up and accumulate heat, and this is the more the closer to the plate. The plate is covered with a heat shell, and the thickness of this shell gradually increases. As a result, heat transfer through it deteriorates. The outer boundary of this thermal shell is associated with a "heat wave" moving at a speed of dx_0/dt , so the representation of such a wave has a physical justification. The number χ easy to calculate analytically, making the inside of the integral the change of variable $\zeta = s^2$, get

$$\chi = \int_0^\infty \frac{e^{-\zeta}}{\sqrt{\zeta}} d\zeta = 2 \int_0^\infty e^{-s^2} ds = 2 \frac{\sqrt{\pi}}{2} = \sqrt{\pi}.$$

And the final expression for the heat transfer coefficient will be

$$\alpha = \frac{1}{\sqrt{\pi}} \frac{\lambda}{\sqrt{\kappa(t + t_*)}}.$$

The corresponding formula (3) for temperature can be recorded via the erf error function:

$$T = T_w - (T_w - T_0) \operatorname{erf}(z), \quad \operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-s^2} ds.$$

If we choose $z_0 \neq 0$, we would get an approximate solution to the problem, and the closer z_0 is to 0, the closer the approximate solution is to the exact solution. The case $z_0 \neq 0$ corresponds to approximate temperature distributions in Fig. 1. It can be seen that in some complex cases (discussed below), when it is not possible to find an exact analytical solution to the problem of thermal conductivity, it is possible to obtain approximate solutions using the idea of "heat wave". One of these cases is the propagation of heat from a cylindrical region into an infinite space, and below we proceed to its detailed analysis.

2. Heat transfer coefficient between a pipe with a constant high temperature and an infinite surrounding cold space.

For cylindrical and spherical problems, the exact self-similar solution cannot be obtained analytically. Therefore, such geometries are of interest for the application of the approximate method, the idea of which is given above. Consider the cylindrical problem below.

Choose a cylindrical coordinate system with the center of symmetry on the pipe axis with radius R , the radial coordinate r is counted from this axis. Along the axis of the pipe, the temperature in it T_p is constant, so for the environment with the initial temperature T_c , the heat equation in the form of

$$\frac{\partial T}{\partial t} = \kappa \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right). \quad (4)$$

In mathematical form, initial and boundary conditions are strictly set in the following form:

$$T(t=0, r) = T_c = \text{const},$$

$$T(t, r=R) = T_p, \quad T(t, r=\infty) = T_c.$$

We find an approximate solution to this problem using the idea of a heat wave. Instead of the given boundary conditions, we take other:

$$T(t, r=r_0(t)) = T_p, \quad T(t, r=\infty) = T_c, \quad (5)$$

where $r_0(t)$ – the front of the thermal wave.

Now we are looking for the solution of equation (4) as a function of one self-similar variable containing the characteristic time $t_* = \text{const}$:

$$T = T(z), \quad z = \frac{r^2}{4\kappa(t+t_*)}.$$

Calculate the derivatives:

$$\frac{\partial T}{\partial t} = \frac{dT}{dz} \frac{\partial r}{\partial t} = -\frac{r}{t+t_*} \frac{dT}{dz}, \quad r \frac{\partial T}{\partial r} = \frac{dT}{dz} \frac{\partial z}{\partial r} = \frac{r^2}{2\kappa(t+t_*)} \frac{dT}{dz},$$

$$\frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) = \frac{r}{\kappa(t+t_*)} \frac{dT}{dz} + \frac{r^2}{2\kappa(t+t_*)} \frac{d^2T}{dz^2} \frac{\partial z}{\partial r} = \frac{r}{\kappa(t+t_*)} \left(\frac{dT}{dz} + z \frac{d^2T}{dz^2} \right).$$

Taking into account the received expressions the equation (4) turns into the ordinary differential equation

$$z \frac{d^2T}{dz^2} + (1+z) \frac{dT}{dz} = 0.$$

Its General solution contains integration constants C1 and C2:

$$T = C_2 - C_1 \int_{z_0}^z \frac{e^{-\zeta}}{\zeta} d\zeta, \quad \text{here } \frac{dT}{dz} = -C_1 \frac{e^{-z}}{z}. \quad (6)$$

The obtained results can be given the following physical interpretation: the pipe is surrounded by a thermal "shell" and its radius varies according to the law $r_0(t)$ (Fig. 2).

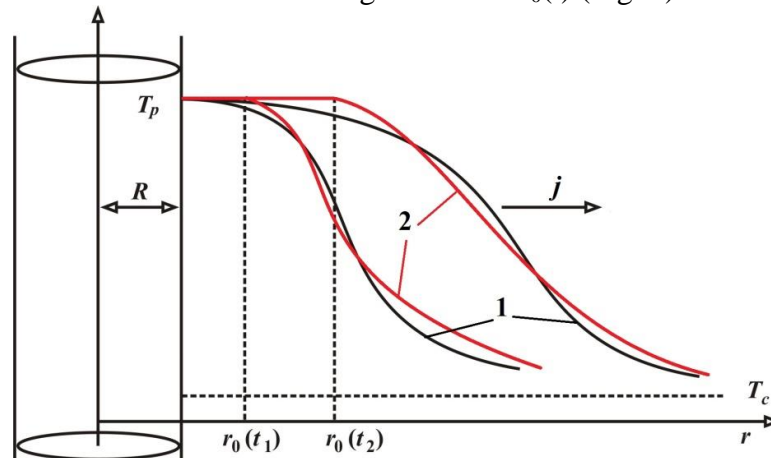


Fig.2. Location of the boundary $r_0(t)$ of the heat wave at two different times t_1 and t_2 :
1 - real temperature profile; 2 - approximate profile.

In the region $r < r_0(t)$ the temperature is constant and equal to T_p , in the region $r > r_0(t)$ the temperature changes according to the first equality in (6). The same figure shows the exact and approximate temperature distributions at different times. Within the shell thickness, the temperature is constant and equal to T_p , this is laid down in the first boundary condition of (5). Therefore, when $z = z_0$, the equality $T = T_p$ must be executed, using the first equality in (6) we find $C_2 = T_p$. The number z_0 remains an indefinite value because of the uncertainty of time t_* , it has the physical meaning of the conditional time when the wave mode of heat propagation begins. But now we can say that the law of changing the thickness of the shell has a specific relationship:

$$r_0(t) = \sqrt{4z_0\kappa(t+t_*)}. \quad (7)$$

The expression for temperature now takes the form

$$T = T_p - C_1 \int_{z_0}^z \frac{e^{-\zeta}}{\zeta} d\zeta. \quad (8)$$

Using the second boundary condition of (5) with this form $T(z)$ we obtain

$$T_0 = T_p - C_1\eta, \quad \eta = \int_{z_0}^{\infty} \frac{e^{-\zeta}}{\zeta} d\zeta,$$

$$\text{or } C_1 = \frac{T_p - T_0}{\eta}.$$

$$\text{Now (8) can be rewritten as } T = T_p - \frac{T_p - T_0}{\eta} \int_{z_0}^z \frac{e^{-\zeta}}{\zeta} d\zeta.$$

Find the heat flow j from the pipe to the environment, this flow must be calculated at the point $r = r_0$:

$$j = -\lambda \frac{\partial T}{\partial r} \Big|_{r=r_0(t)} = -\lambda \frac{\partial T}{\partial z} \Big|_{z=z_0} \frac{\partial z}{\partial r} \Big|_{r=r_0(t)} = \frac{\lambda}{\eta\sqrt{\kappa(t+t_*)}} \frac{\exp(-z_0)}{\eta\sqrt{z_0}} (T_p - T_0).$$

The second equation of (6) is used to calculate the temperature derivative. From here we can see that the heat transfer coefficient for the pipe is

$$\alpha = \frac{\lambda}{\eta\sqrt{\kappa(t+t_*)}} \frac{\exp(-z_0)}{\eta\sqrt{z_0}}. \quad (9)$$

Thus, the formula for the heat transfer coefficient with two uncertain parameters η and t_* is obtained. It cannot be calculated only from the analysis of equations (8) and (9), to determine the η and t_* need an accurate solution to the problem. The exact solution is assumed to be obtained by numerical solution of equation (4) with reduced boundary and initial conditions (5).

3. Comparison of approximate solutions with the results of numerical solution of the problem of heat propagation from a heated pipe to an unlimited cold space.

The following problem was solved numerically to determine the parameters η and t_* and to check the accuracy of the obtained result for the heat transfer coefficient (9)

$$\frac{\partial T}{\partial t} = \kappa \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right), \quad (10)$$

Boundary and initial conditions:

$$T|_{r=R} = T_p; \quad \left. \frac{\partial T}{\partial r} \right|_{r \rightarrow \infty} = 0; \quad T|_{t=0} = T_c.$$

Here κ – coefficient of thermal diffusion: $\kappa = \lambda/(c\rho)$, where λ – coefficient of thermal conductivity; c – capacity; ρ – density.

This problem simulates the propagation of heat from a pipe of radius R to an infinite space, in which at the initial time the temperature was T_c , the temperature of the pipe is constant and equal to T_p (Fig. 2). In formula (7), the numerical values of the parameters η and t_* remain undefined. To determine them, it is necessary to compare the heat fluxes determined in the numerical solution and in the approximate way according to the formula (12) below. The condition of maximum proximity of these heat fluxes is used to determine η and t_* . An implicit scheme [4, 5] was used for the numerical solution. In the numerical solution, the heat flux on the pipe surface is determined by the equation

$$j_0 = -\lambda \frac{T_1 - T_0}{h}, \quad (11)$$

where T_1 and $T_0 = T_p$ – are the temperature values at the nodes respectively with numbers $i = 1$ and $i = 0$.

In parallel, together with the definition (11), the calculation of the heat flow is carried out according to the formula

$$j_t = \alpha(T_p - T_c), \quad \alpha = \frac{\lambda}{\sqrt{\kappa(t + t_*)}} \frac{\exp(-z_0)}{\eta \sqrt{z_0}}. \quad (12)$$

Here the numerical values of the parameters η and t_* are chosen so that the heat flux j_t coincides with the calculated flux j_0 by the formula (11).

After finding η and t_* required to carry out the evaluation of the integral

$$\eta = \int_{z_0}^{\infty} \frac{e^{-\zeta}}{\zeta} d\zeta, \quad (13)$$

where the lower limit of integration z_0 must be chosen such that the calculated integral coincides with the previously found value η .

The calculations are performed with the following physical parameters: $R = 0.1$ m, $\lambda = 1.2$ W/(m·K), $\rho = 3000$ kg/m³, $c = 1200$ J/(kg·K), these data give the ratio $\kappa = 3.33 \cdot 10^{-7}$ m²/s. The ambient temperature is $T_c = -5$ °C, the temperature on the pipe surface $T_p = 117$ °C. The values $\eta = 1.828$; $t_* = 50$ s; $z_0 = 0.8 \cdot 10^{-5}$.

For Fig.3 the values of the heat flux from the pipe calculated by the formula (12) and its exact value by the results of the numerical solution of the equation (10) are given. The time on the horizontal axis is given in hours. As shown in Fig. 3 the values of the heat flux in the two methods of calculating the heat flux are very different only at the beginning for a few tens of seconds. In the subsequent time difference is weak.

The change in the radius of the heated zone $r_0(t)$ in time is shown in Fig. 4. Knowledge of $r_0(t)$ does not give much practical use, because we need to know the heat flow to calculate the heat loss from the pipeline. The radius $r_0(t)$ is mainly of theoretical interest.

The temperature distribution $T(t, r)$ is shown in Fig. 5 by results of numerical solution of equation (10). Due to the low coefficient of thermal conductivity, the heating of the surrounding pipeline is slow, more than nine hours the heat wave has shifted by less than 0.8 mm.

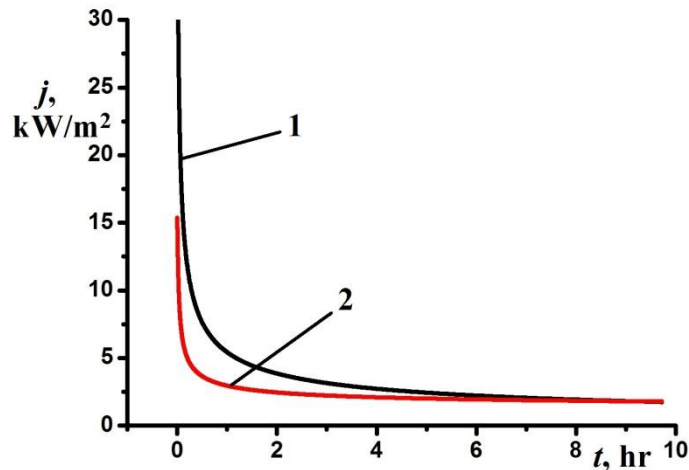


Fig.3. Dynamics of heat flow calculated by the formula (12) - (j_0 , curve 1), and from the result of numerical solution of equation (10) (j_t , curve 2).

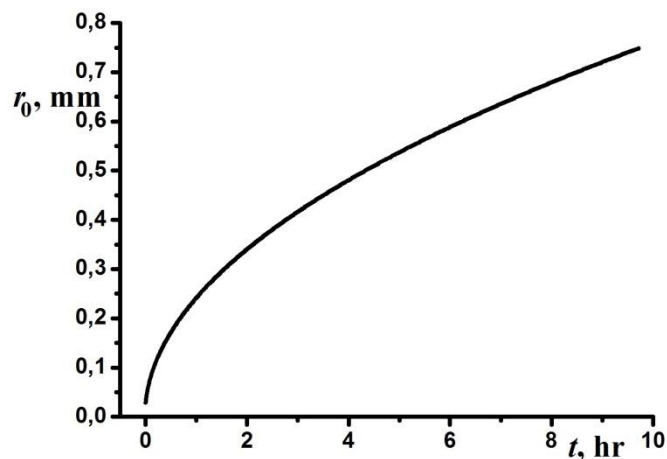


Fig.4. The dependence of the radius $r_0(t)$ on time.

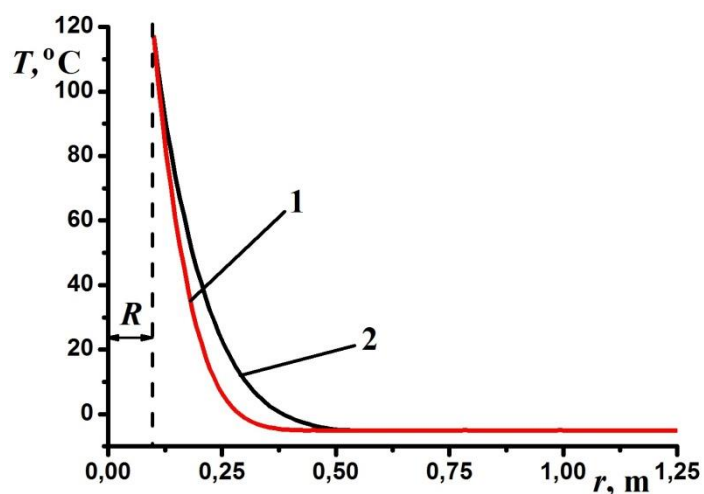


Fig.5. Numerical results of determining the temperature distribution in the vicinity of the pipe at different times: cu. 1 - $t = 4.17$ hours; cu. 2 - $t = 9.72$ hours.

Conclusion

Let us explain the physical nature of the decrease in the heat transfer coefficient: due to the large temperature difference in the pipe and the surrounding space, at the initial time there is a powerful heat transfer, but as the medium warms up near the pipe, a heated layer is gradually formed around it and the layer thickness gradually grows. As a result, around pipe a thermal "cushion" is created, the heat from the pipe must pass through this "cushion" before entering the cold area of the environment. Thus, the increasing heat "cushion" leads to blocking the heat loss from the pipe, which means a decrease in the heat transfer coefficient.

The main result of the study is a simple formula (12), taking into account the variability of the heat transfer coefficient. It is very convenient to use when carrying out engineering calculations of heat loss in thermal networks, and the results obtained on its basis will allow solve more precisely problems of design and optimization of thermal networks.

REFERENCES

- 1 Tkachenko L.A., Repin A.V. *Theory of heat transfer*. Kazan, Kazan University Publishing House, 2017, 151 p. [in Russian].
- 2 Crane F., Black W. *Heat Transfer Basics*. Moscow, World, 1983, 510 p.
- 3 Tikhonov A.N., Samarsky A.A. *Equations of mathematical physics*. Moscow, Science, 1977, 736 p. [in Russian]
- 4 Kalitkin N.N. *Numerical methods. Training allowance*. SPb, BHV-Petersburg, 2011, 512 p. [in Russian]
- 5 Sabdenov K.O., Erzada M. *Mathematical modeling of systems and processes*. Astana, Publishing House of Gumilyov ENU, 2014, 256 p. [in Russian]