UDK 532: 529.5

MODELING OF A TWO-PHASE FLOW OF LIQUID WITH SMALL-SIZE GAS BUBBLES

Vasenin I.M., Narimanov R.K., Shrager L.A., Perchatkina E.V.

Tomsk State University, Tomsk, Russia, ring_0@mail.ru

The model of the motion of a gas-liquid medium with small-size bubbles in a gravity field following free and forced convection was proposed. The model automatically takes into account the processes causing free convection in gravity field in the presence of heterogeneous concentration of bubbles. Compared to the model of interpenetration continuums to describe a two-phase medium, this model does not contain small parameters for derivatives. The two-phase flow in context of the problems similar to the water ozonation problem in contact tanks is considered. The analogy to compressible gas models allows obtaining the solution using well-established numerical solution schemes.

Keywords: two-phase flow, gas-liquid medium, small-size bubbles, heterogeneous, water ozonation

Introduction

The movement of gas-liquid mixtures is wide-spread in nature and various technological processes. The diversity of these phenomena is determined by the proceeding physicochemical reactions, external conditions, the object dimensions, the magnitude of gas phase bubbles, etc. One of the actual technological processes of increasing interest is the disinfection of water in water-supply systems through its ozonation [1-4]. This is due to the high activity of ozone, which allows you effectively affect many types of pollution, both natural and artificial origin. The other purifying methods, for example, electropulse water machining [5], are also of some interest.

The complex motion of multiphase media stimulates the development of methods for sensor monitoring of the gas-liquid mixture behavior [6]. From the great number of the works devoted to the experimental study of ascending gas-liquid flows, we should take note of the following [7–9]. The works devoted to the numerical simulation of two-phase bubble motion are very numerous and are mainly held by the Euler continual approach of representing polydisperse flows [10–13]. The authors in their works [12–13] suggest the promising method for computing the bubble velocities by solving transport equations. Equations of water-air mixtures in context of various problems are periodically considered in the scientific literature. However, the mixture flow in gravity field, taking into account combined action of free and forced convection, has not been practically studied.

Thus, the urgency of the development of the gas-liquid mixture models is undeniable. The purpose of this work is to develop a physico-mathematical model of a two-phase flow of water with small-size bubbles in context of the problems similar to the problem of water ozonation in contact tanks.

1. Physical task description

In this approach, the movement of bubbles in water is considered as the movement of small particles with a density much lower than the density of water. The physical basis of this convection is very simple. By means of the Archimedes buoyant force a lighter mixture containing a larger amount of gas floats in a heavier fluid in the same way as light warm air floats in a cold environment. The mathematical model for small-size bubbles is simpler than the equations of two-phase fluid flow with arbitrary size bubbles and allows us to significantly simplify the problem.

We will call air bubbles small if for their sizes the following conditions are satisfied: the constancy of the bubble shape; the equality of the temperature of the bubble to the ambient

.

temperature; the short bubble velocity setting time. Moving in water bubbles retain their spherical shape by virtue of the action of surface tension force, so that the movement of each bubble can be considered as the movement of a spherical particle. To satisfy this requirement bubbles, floating up in water under the action of Archimedes buoyant force, should have a size smaller than $2-3\times10^{-3}m$ [14]. The estimates [15] show that for air bubbles with a diameter less than $2.5\cdot10^{-3}m$ the relaxation time of the air and water temperature difference does not exceed 0.1 *s*. Since the time of small-size bubble staying in the water mass of a contact tank is assumed to be quite long, the temperature difference between bubbles and water can be neglected. The transient time of the velocity of bubbles floating up in water should be negligible compared with the characteristic time of their motion in the whole region. Since this requirement is essential for further simplifications, we will address this issue in more detail.

Consider the equation of the gravitational small-size bubble floating-up in a liquid medium.

$$\frac{d\mathbf{U}_s}{dt}=\frac{C_R}{m}(\mathbf{U}-\mathbf{U}_s)+\mathbf{g}(1-\frac{\rho_{в}^{0}}{\rho_S^0}), \tag{1}$$

where $\mathbf{U}_S$, ρ_S^0 – bubble velocity and the density of gaseous phase; $\mathbf{U}$, $\rho_{в}^{0}$ – the velocity and density of water, t – time, m – bubble mass, C_R – the resistance coefficient of moving bubble. In general, the C_R variable is: $C_R=\frac{1}{2}C_D\rho_{в}^{0}S\left|\mathbf{U}-\mathbf{U}_S\right|$, where C_D – resistance coefficient depending on the Reynolds number, S – bubble mid-section area.

Taking into account the added masses, the equation of particle motion is written as

$$\left(\frac{1}{2}+\frac{\rho_s}{\rho_в}\right)\frac{\tau^*}{T_0}\frac{d\left(\vec{u}-\vec{u_s}\right)}{d\bar{t}}=\left(\frac{1}{2}+\frac{\rho_s}{\rho_в}\right)\frac{\tau^*}{T_0}\frac{d\left(\vec{u}\right)}{d\bar{t}}-f_R\left(\left|\vec{u}-\vec{u_s}\right|\right)\left(\vec{u}-\vec{u_s}\right)-\tau^*\vec{g}\left(1-\frac{\rho_s}{\rho_в}\right)$$

where $\tau^*=\frac{m}{C_R}$ – the relaxation time of bubble velocity, i.e. $\tau^*=d_s^2\rho_s^0/18\mu$, T_0- the characteristic time of bubble floating-up in ozonator; $\bar{t}=\frac{t}{T_0}$; μ - dynamic coefficient of viscosity for water, $f_R\left(\left|\vec{u}-\vec{u}_s\right|\right)=(1+0{,}15\,\mathrm{Re}^{0{,}682})$.

External decomposition of the solution is sought by the method of successive approximations as $\vec{u}-\vec{u_s}=\vec{y_0}(\bar{t})+\frac{\tau^*}{T_0}\vec{y_1}(\bar{t})$. This takes into account that outside the initial boundary layer, $t>\frac{\tau^*}{T_0}$, the variable $\frac{d\vec{u}_s}{d\bar{t}}$ has the order $\frac{|\vec{u}_s|}{|1|}\approx0{,}2\,m/\mathrm{s}$, while the product is $gT_0\approx200\,m/\mathrm{s}$. Writing the equation in the projections, we obtain the system of equations:

$$\left(\frac{1}{2}+\frac{\rho_s}{\rho_в}\right)\frac{\tau^*}{T_0}\frac{d\left(y_{0x}+\frac{\tau^*}{T_0}y_{1x}\right)}{d\bar{t}}-\left(\frac{1}{2}+\frac{\rho_s}{\rho_в}\right)\frac{\tau^*}{T_0}\frac{du_x}{d\bar{t}}+f_R\left(\left|\vec{u}-\vec{u_s}\right|\right)\left(y_{0x}+\frac{\tau^*}{T_0}y_{1x}\right)=\tau^*g\left(1-\frac{\rho_s}{\rho_в}\right)$$

$$\left(\frac{1}{2}+\frac{\rho_s}{\rho_в}\right)\frac{\tau^*}{T_0}\frac{d\left(y_{0y}+\frac{\tau^*}{T_0}y_{1y}\right)}{d\bar{t}}-\left(\frac{1}{2}+\frac{\rho_s}{\rho_в}\right)\frac{\tau^*}{T_0}\frac{du_y}{d\bar{t}}+f_R\left(\left|\vec{u}-\vec{u_s}\right|\right)\left(y_{0y}+\frac{\tau^*}{T_0}y_{1y}\right)=0$$

where $\left|\vec{u}-\vec{u}_s\right|=\sqrt{(u_x-u_{xs})^2+(u_y-u_{ys})^2}$.

From the second equation of the system, taking into account that $f_R\left(\left|\vec{u}-\vec{u}_s\right|\right)\neq 0$ and, rejecting the terms of the order $\frac{\tau^*}{T_0}$, we can obtain that $y_{0y}=0$ and therefore in this approximation $f_R\left(\left|\vec{u}-\vec{u}_s\right|\right)=f_R\left(\left|u_x-u_{sx}\right|\right)$. Rejecting in the first equation the summands of order $\frac{\tau^*}{T_0}$ and taking into account the remark about the value of the right-hand side, we obtain that in this approximation the term characterizing the resistance force takes the form. In the first equation rejecting the summands of the order $\frac{\tau^*}{T_0}$ and taking into account the remark about the value of the equation right-hand side, we obtain that in this approximation the term characterizing the resistance force takes the following form $f_R(|y_{0x}|(y_{0x}))=(1-\frac{\rho_s}{\rho_в})\tau g$.

Assuming that the air density $\rho_s^0=1.29\ \ kg/\mathrm{m}^3$, the dynamic coefficient of viscosity for water $\mu=10^{-3}\ \ kg/\mathrm{m}\cdot s$ and the diameter of bubbles $d_S=2\times10^{-3}m$ for the Stokes resistance law, we can obtain that $\tau^*=d_s^2\rho_s^0\big/18\mu\approx 3\times10^{-4}$ s. However, when evaluating it is necessary to take into account the added mass of water, which is several times the mass of air in a bubble. Hence instead of $\tau^*=3\cdot10^{-4}$ s, we obtain the value $\tau^*=1.2\cdot10^{-1}$ s. This time is two orders of magnitude shorter than the time T_0 (which is 10-30 s) of bubble floating-up in the 4 m high ozonator. Thus, the relaxation time of the velocity of bubbles with a diameter of $\sim 2\times10^{-3}m$ is far less than the characteristic time of the process under consideration, so to simplify the solution we can apply the asymptotic expansion in the small parameter $\tau=\tau^*\big/T_0$. As obtained above we introduce

$$\mathbf{F}=\mathbf{g}\left(1-\frac{\rho_в^0}{\rho_S^0}\right).$$

In the agreed notation, the equation (1) is written in the following form

$$\frac{d\mathbf{U}_s}{dt}=\frac{1}{\tau}\mathbf{U}-\frac{1}{\tau}\mathbf{U}_s+\mathbf{F}. \tag{2}$$

From this equation, the bubble velocity $\vec{U}_S$ can be expressed analytically and we can obtain an integral equation having the form

$$\mathbf{U}_s=e^{-\int_0^t\frac{dt}{\tau}}\left[\mathbf{U}_{s0}+\int_0^t(\frac{1}{\tau}\mathbf{U}+\mathbf{F})e^{\int_0^\tau\frac{dt}{\tau}}dt\right]. \tag{3}$$

Integrating by parts and leaving only the first order terms on τ, we will obtain

$$\mathbf{U}_s=\mathbf{U}+\tau\mathbf{F}-\tau\frac{d\mathbf{U}}{dt}+e^{-\int_0^t\frac{dt}{\tau}}\left[\mathbf{U}_{s0}-(\mathbf{U}+\tau\mathbf{F})_0+\tau\frac{d\mathbf{U}}{dt}\Big|_0\right]+\theta(\tau^2), \tag{4}$$

where index "0" is for the initial values and the value $\theta\left(\tau^2\right)$ denotes second-order terms of smallness.

.

2. The mathematical modeling of a two-phase flow with small-size bubbles of gas mixture

Note that in formula (4) the time derivative is taken along the bubble trajectory, that is

$$\frac{d\mathbf{U}}{dt} = \frac{\partial \mathbf{U}}{\partial t} + \mathbf{U}_s \nabla \mathbf{U}.$$

Further, mindful that the relaxation time of bubble velocity τ is small, in (4) we dismiss the terms of the second order in τ. As follows from (4), after the relaxation time expires, the initial conditions can also be ignored. As a result of simple transformations over the remaining terms of the equation, we obtain

$$m\left(\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U}_s \nabla \mathbf{U}\right) \approx \frac{1}{2} C_D S \rho_{\text{в}}^0 |\mathbf{U} - \mathbf{U}_s| (\mathbf{U} - \mathbf{U}_s) + m\mathbf{g}(1 - \frac{\rho_{\text{в}}^0}{\rho_S^0}). \quad (5)$$

Being rewritten this equation assumes the form of

$$m\left(\frac{\partial \mathbf{U}}{\partial t} + \left[\mathbf{U} + (\mathbf{U}_s - \mathbf{U})\right] \nabla \mathbf{U}\right) \approx \frac{1}{2} C_D S \rho_{\text{в}}^0 |\mathbf{U} - \mathbf{U}_s| (\mathbf{U} - \mathbf{U}_s) + m\mathbf{g}(1 - \frac{\rho_{\text{в}}^0}{\rho_S^0}), \quad (6)$$

It can be easily discerned that from (6) for a given fluid velocity field all the projections of the vector $(\mathbf{U} - \mathbf{U}_s)$ for the bubble velocity lag can be calculated by solving a system of algebraic equations.

When deriving the equations of two-phase convection in water with small-size bubbles, for the sake of simplicity we assume that the two-phase convective flow is two-dimensional and laminar. The axis OY is vertically guided in the direction opposite to the direction of gravity. In particular, in the case of the Stokes resistance law $(C_D = 24/\mathrm{Re})$ for a two-dimensional flow in x, y plane with velocities $\mathbf{U} = (u, v)$, $\mathbf{U}_s = (u_s, v_s)$, approximately for small τ to within the terms of the order τ^2, we get

$$u - u_S \approx \tau \frac{\partial u}{\partial t}, \quad (7)$$

$$v - v_S \approx \tau \left[\frac{\partial v}{\partial t} - g\left(1 - \frac{\rho_{\text{в}}^0}{\rho_S^0}\right)\right]. \quad (8)$$

where $\tau = d_s^2 \rho_s^0 \big/ 18\mu T_0$ – the relaxation time; d_s – bubble diameter; μ – liquid viscosity coefficient.

Let us denote the bubble velocity vector as $\mathbf{U}_s = (u_s, v_s)$ and the water velocity vector as $\mathbf{U} = (u, v)$. Let $\rho_l = \rho_{\text{в}}^0 (1 - \rho_s / \rho_s^0)$ denote the mass of liquid in a unit volume, ρ_s – the mass of the bubbles in a unit volume, p – pressure. Taking into account the conditions made, we write down the projections of the two-phase mixture motion equations on the x and y axes.

$$\begin{aligned} &\rho_l \left(\frac{\partial u}{\partial t} + \mathbf{U} \nabla u\right) + \rho_S \left(\frac{\partial u_S}{\partial t} + \mathbf{U}_S \nabla u_S\right) + \frac{\partial p}{\partial x} = \mu \Delta u, \\ &\rho_l \left(\frac{\partial v}{\partial t} + \mathbf{U} \nabla v\right) + \rho_S \left(\frac{\partial v_S}{\partial t} + \mathbf{U}_S \nabla v_S\right) + \frac{\partial p}{\partial y} = \mu \Delta v - g(\rho_l + \rho_S), \end{aligned} \quad (9)$$

Introduce new variables $\delta u_s = u_s - u, \quad \delta v_s = v_s - v$ and vector $\mathbf{W} = (\delta u_s, \delta v_s)$. The system of equations (9) can be rewritten with the new variables

$$\begin{aligned}
&(\rho_l+\rho_S)\left(\frac{\partial u}{\partial t}+\mathbf{U}\nabla u\right)+\rho_S\left(\frac{\partial \delta u_S}{\partial t}+\mathbf{W}_S\nabla u+\mathbf{U}_S\nabla\delta u_S\right)+\frac{\partial p}{\partial x}=\mu\Delta u,\\
&(\rho_l+\rho_S)\left(\frac{\partial v}{\partial t}+\mathbf{U}\nabla v\right)+\rho_S\left(\frac{\partial \delta v_S}{\partial t}+\mathbf{W}_S\nabla v+\mathbf{U}_S\nabla\delta v_S\right)+\frac{\partial p}{\partial y}=\mu\Delta v-g(\rho_l+\rho_S).
\end{aligned} \tag{10}$$

Assume that

$$\bar{u}=\frac{u}{V},\ \bar{v}=\frac{v}{V},\ \overline{u_S}=\frac{u_S}{V},\ \overline{v_S}=\frac{v_S}{V},\ \bar{\mathbf{U}}=\left(\frac{u}{V},\frac{v}{V}\right),\ \bar{\mathbf{W}}_S=\left(\frac{\delta u_S}{\delta V},\frac{\delta v_S}{\delta V}\right),\ \overline{\delta u_S}=\frac{\delta u_s}{\delta V},\ \overline{\delta v_S}=\frac{\delta v_s}{\delta V},$$

$$\bar{\mathbf{U}}_S=\left(\overline{u_S},\overline{v_S}\right),\ \overline{\rho_l}=\frac{\rho_l}{\rho_l^0},\ \overline{\rho_S}=\frac{\rho_S}{\rho_l^0},\ \overline{p}=\frac{p}{\rho_l^0V^2},\ \bar{x}=\frac{x}{L},\ \bar{y}=\frac{y}{L},\ \bar{t}=\frac{t}{(L/V)},$$

where L – the length scale, V – the velocity scale, δV – the scale of bubble velocity lag.

The equations (10) can be transformed to the non-dimensional form

$$\begin{aligned}
&\left(\frac{\partial\bar{u}}{\partial t}+\bar{\mathbf{U}}\nabla\bar{u}\right)+\frac{\overline{\rho_S}}{\overline{\rho_l}+\overline{\rho_S}}\frac{\delta V}{V}\left(\frac{\partial\overline{\delta u_S}}{\partial t}+\bar{\mathbf{W}}_S\nabla\bar{u}+\bar{\mathbf{U}}\nabla\overline{\delta u_S}\right)+\frac{1}{\overline{\rho_l}+\overline{\rho_S}}\frac{\partial\bar{p}}{\partial\bar{x}}=\frac{1}{\mathrm{Re}}\Delta\bar{u},\\
&\left(\frac{\partial\bar{v}}{\partial t}+\bar{\mathbf{U}}\nabla\bar{v}\right)+\frac{\overline{\rho_S}}{\overline{\rho_l}+\overline{\rho_S}}\frac{\delta V}{V}\left(\frac{\partial\overline{\delta v_S}}{\partial t}+\bar{\mathbf{W}}_S\nabla\bar{v}+\bar{\mathbf{U}}\nabla\overline{\delta v_S}\right)+\frac{1}{\overline{\rho_l}+\overline{\rho_S}}\frac{\partial\bar{p}}{\partial\bar{z}}=\frac{1}{\mathrm{Re}}\Delta\bar{v}-\frac{1}{\mathrm{Fr}}.
\end{aligned} \tag{11}$$

Here $\mathrm{Re}=\dfrac{(\rho_l+\rho_s)VL}{\mu}$ – Reynolds number, $\mathrm{Fr}=\dfrac{V^2}{gL}$ – Froude number.

From equations (11) it can be seen that in the region of large Reynolds numbers when the inequality (12) is satisfied

$$\frac{\overline{\rho_S}}{\overline{\rho_l}+\overline{\rho_S}}\frac{\delta V}{V}<<1, \tag{12}$$

we can neglect the second terms as compared with the first ones in the left part of these equations, and solve the following system

$$\begin{aligned}
&\frac{\partial\bar{u}}{\partial t}+\bar{\mathbf{U}}\nabla\bar{u}+\frac{1}{\overline{\rho_l}+\overline{\rho_S}}\frac{\partial\bar{p}}{\partial\bar{x}}=\frac{1}{\mathrm{Re}}\Delta\bar{u},\\
&\frac{\partial\bar{v}}{\partial t}+\bar{\mathbf{U}}\nabla\bar{v}+\frac{1}{\overline{\rho_l}+\overline{\rho_S}}\frac{\partial\bar{p}}{\partial\bar{y}}=\frac{1}{\mathrm{Re}}\Delta\bar{v}-\frac{1}{\mathrm{Fr}}.
\end{aligned} \tag{13}$$

Near the solid boundaries, due to the no-slip condition, the velocity of liquid along with inertial terms tends to zero. Meanwhile under the influence of Archimedes force bubbles continue to float up. Consequently, in the equations (11) near the solid walls, due to the presence of terms of the form $\bar{\mathbf{W}}_s\nabla\bar{u}$ and $\bar{\mathbf{W}}_s\nabla\bar{v}$, corrections to the inertial terms can become comparable.

In these cases, when fluid moves, viscous force begins to play a defining role. Therefore, the value of the terms in equations (11) should be evaluated in comparison with the viscous terms. It is easy to see that in the region of small Re numbers, the terms dropped above will be small if the inequality (14) is satisfied.

$$\frac{\overline{\rho_S}}{\overline{\rho_l}+\overline{\rho_S}}\frac{\delta V}{V}<<\frac{1}{\mathrm{Re}}=\frac{\mu}{(\overline{\rho_l}+\overline{\rho_S})VL}. \tag{14}$$

With simultaneous satisfaction of the inequalities (12) and (14), equations (13) can be used in the entire flow region. Both inequalities coincide in form, if for the length scale in (14) we choose the distance L, on which the Reynolds number is equal to 1 near the solid boundary. However, the

.

sense of these inequalities is completely different and they are satisfied for the different scales. In the inequality (12), parameters V and L refer to the flow region with the large Reynolds numbers, and the inequality (14) includes the scales characterizing the flow region for the small Reynolds numbers. To close the system of equations (13), we should add the mass conservation equation for the two-phase mixture, the bubble mass conservation equation, the energy equation, and the equation of state.

3. Equation of state of a two-phase mixture containing water and gas bubbles

Let $\rho = \rho_l + \rho_s$ denote the mixture mass density, z - water mass fraction in the mixture, $z = \frac{\rho_l}{\rho}$. Then $\frac{\rho_s}{\rho} = (1-z)$. The mass unit of the mixture, which occupies specific volume $\frac{1}{\rho}$, contains z kg of water, which occupies the volume $\frac{z}{\rho_в^0}$, and $(1-z)$ kg of gas, which occupies the volume $\frac{1-z}{\rho_s^0}$. Owing to the additivity of the volumes, we can write

$$\frac{1}{\rho} = \frac{z}{\rho_в^0} + \frac{(1-z)}{\rho_s^0}. \tag{15}$$

Express from (15) the value $\frac{1}{\rho_s^0}$ and substitute it into the ideal gas equation of state $\frac{P}{\rho_s^0} = \frac{RT}{\mu_s}$, where μ_s – the molar mass of gas. As a result we obtain

$$P\left(\frac{1}{\rho} - \frac{z}{\rho_в^0}\right) = (1-z)\frac{R}{\mu_s}T. \tag{16}$$

At the pressure up to 10^8 Pa, the dependence of water density on pressure is described by the experimentally obtained linear law [15]

$$\rho_в^0 = \rho^*\left(1 + \frac{P}{k}\right), \tag{17}$$

where ρ^* and k is the medium parameters.

Substituting (17) into (16) and defining a variable $\bar{R} = \frac{(1-z)R}{\mu_s}$ we obtain the mixture equation of state (18)

$$P\left[\frac{1}{\rho} - \frac{z}{\rho^*\left(1+\frac{P}{k}\right)}\right] = \bar{R}T. \tag{18}$$

4. Energy equation for a two-phase mixture containing water and small-size bubbles

Since the temperature of water and small-size bubbles can be considered equal, the amount of heat that is contained in the mixture with a constant volume is

$$\bar{C}_V \rho T = z\rho C_в T + (1-z)\rho C_V T, \tag{19}$$

where $C_в$ – heat capacity of water, C_V – heat capacity of gas at constant volume.

From here it follows that the heat capacity of the mixture $\overline{C}_V$ is

$$\overline{C}_V = zC_{\text{в}} + (1-z)C_V. \tag{20}$$

Similarly, for the heat capacity of the mixture at constant pressure, we can find that

$$\overline{C}_p = zC_{\text{в}} + (1-z)C_p. \tag{21}$$

Ratio of specific heat of this mixture is

$$\overline{k} = \frac{\overline{C}_p}{\overline{C}_V} = \frac{zC_{\text{в}} + (1-z)C_p}{zC_{\text{в}} + (1-z)C_V}. \tag{22}$$

Thus, a two-phase mixture of water and small-size bubbles can be considered as a “gas” with the heat capacities (20), (21), ratio of specific heat (22), and the equation (18). Let us calculate the entropy of this "gas". Firstly, it should be noted that the equation of state (18) resembles the Van der Waals equation of state, if in the latter we neglect molecular collisions.

According to [16] the entropy of this gas written in the above terms is

$$\overline{S} = C_v \ln T + \left(\frac{1}{\rho} - \frac{z}{\rho^* \left(1 + \frac{P}{k}\right)} \right) \overline{R}, \tag{23}$$

in the adiabatic process remains constant.

Therefore, from (23) setting $\frac{\overline{S}}{\overline{R}}$ equal to some constant value we can get

$$\left[\frac{1}{\rho} - \frac{z}{\rho^* \left(1 + \frac{P}{k}\right)} \right] T^{\frac{\overline{C}_V}{\overline{R}}} = \left[\frac{1}{\rho_0} - \frac{z}{\rho^* \left(1 + \frac{P_0}{k}\right)} \right] T_0^{\frac{\overline{C}_V}{\overline{R}}}.$$

Hence, using the equation of state (18) and taking into consideration the equality $\overline{C}_P - \overline{C}_V = \overline{R}$ we find

$$\frac{T}{T_0} = \left(\frac{P}{P_0} \right)^{\frac{\overline{k}-1}{\overline{k}}}. \tag{24}$$

We use equation (24) to evaluate the variation of the adiabatic temperature of a two-phase medium upon changing pressure. Put the case that the mass fraction of bubbles z in the contact tank does not exceed 10^{-4}. Taking this estimate as a basis, we find that the mass fraction of water is $(1-z) \sim 0.9999$. Assuming that the heat capacity of water $C_{\text{в}}$=4.180 $\frac{kJ}{kg \cdot K}$, we get $\overline{k}$ = 1.000007.

Substituting pressure ratio $\frac{P}{P_0}$ = 2 into the formula (24), we find that with this pressure change, the temperature of the two-phase medium in the tank changes by 0.0005%. It follows as a logical consequence that the two-phase medium consisting of water and gas bubbles will have an almost constant temperature while moving in a tank. Also knowing that the temperature of the phases according to the above almost coincides, further it is possible to consider them identical and equal to a certain value T_0. This value, equal to the temperature of the incoming water, will be used instead of the energy equations of a two-phase mixture and bubbles.

.

Conclusion

The mathematical model formulated above, unlike the model of interpenetrating continuums for a two-phase medium, does not contain small parameters for derivatives and is much simpler from the point of view of numerical solution. By virtue of the consideration of the medium compressibility and the dependence of the density on the concentration of bubbles, this model automatically takes into account the processes causing free convection in the gravity field in the presence of the heterogeneous concentration of bubbles. This kind of convection, when the supply of gas mixture is not uniform in space, significantly affects the duration of stay of the bubbles in the reactor and, consequently, the completeness of the reactions that occur. An additional advantage of the proposed mathematical model is its analogy with compressible gas models. Going forward this analogy makes it possible to use well-developed numerical schemes for solving equations of gas dynamics.

Notwithstanding the fact that for simplicity the above conditions were presented on the example of two-dimensional equations for a two-phase mixture, they remain valid in the case of three-dimensional flows.

REFERENCES

1 Kuzubova L.I., Kobrina V.N. *Chemical methods of water preparation. Chlorination, ozonation, fluoridation.* Novosibirsk, 1996, 132 p. [in Russian]

2 Kozhinov V.F., Kozhinov I.V. *Water ozonation.* Moscow, Stroiizdat, 1974, 496 p. [in Russian]

3 Orlov V.A. *Water ozonation*. Moscow, Stroiizdat, 1984, 88 p. [in Russian]

4 Dankverts P.V. *Gas-liquid reactions*. Moscow, Himia, 1973. [in Russian]

5 Shaimerdenova K.M., Aitpaeva Z.K., Hasenov A.K., Kutum B.B.,Bulkairova G.A. Changes in the structure of the electrodes and their destruction during the electric pulse water treatment. *Eurasian Physical Technical Journal***.** 2017, Vol. 14, No. 1(27), pp. 103 – 108.

6 Pribaturin N.A., Meledin V.G. Multi-informative methods of experimental study of the structure and evolution of two-phase flows. *Eurasian Physical Technical Journal*. 2013, Vol. 10, No. 2(20), pp. 3 – 10.

7 Burdukov A.P., Valukina N.V., Nakoryakov V.E. Features of the flow of a gas-liquid bubble mixture at low Reynolds numbers. *Prikladnaya Mehanika & Tehnicheskaya Fizika - Journal of Applied Mechanics and Technical Physics.* 1975, No. 4, pp. 137-141. [in Russian]

8 Hibiki T., Coda H., Kim S. et al. Structure of Vertical Downward Bubbly Flow. *Int. J. Heat Mass Transfer*. 2004, Vol. 47, pp. 1847 – 1862.

9 Kashinski O.N., Randin V.V., Lobanov P.D., Bogoslovtsev G.V. Influence of the gas phase dispersion on the characteristics of a bubbly flow. *Teplofizika & Aeromehanika.* 2005, Vol. 12, No. 4, pp.637 – 643. [in Russian]

10 Ramkrishna D. *Population Balances. Theory and Applications to Particulate Systems in Engineering*. N.Y., Acad. Press, 2000, 355 p.

11 Marchisio D.L., Fox R.O. *Computational Models for Polydisperse Particulate and Multiphase Systems.* Cambridge, Cambridge University Press, 2013? 508 p.

12 Terekhov V.I., Pahomov M.A. Influence of bubbles on the flow structure and friction in a descending turbulent gas-liquid flow. *Teplofizika & Aeromehanika.* 2008, Vol. 15, No. 4, pp. 629 – 642. [in Russian]

13 Pahomov M.A., Terekhov V.I. Simulation of a turbulent flow structure in an ascending polydisperse gas-liquid flow. *Izvestiya RAN. Mekhanika zhidksoti i gaza*. 2015, No. 2, pp. 57 – 69. [in Russian]

14 Kraiko A.N., Nigmatulin R.I., Starkov L.E., Sternin L.E. et al. Mechanics of multiphase media. Results of science and technology. *Hydromechanics*. Moscow, 1972, Vol. 6, pp. 94. [in Russian]

15 Parkin B.R., Gilmor F.R., Broud G.L. *Shock waves in water with air bubbles.* In the coll. *Underwater and underground explosions*. Moscow, Mir, 1974, pp. 152 – 258. [in Russian]

16 Landau L.D., Lifshitz E.M. *Statistical physics*. Part. 1. Moscow, Nauka, 1976, 584 p. [in Russian]

Article accepted for publication 05.06.2019