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**CREATION AND DEVELOPMENT OF THE FUNDAMENTAL AREA
"FRACTAL RADIOPHYSICS AND FRACTAL RADIO ELECTRONICS:
DEVELOPMENT OF FRACTAL RADIO SYSTEMS".
PART 1. THEORY AND MAIN SCIENTIFIC PROSPECTS.**

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In the first part of the article, scientific prospects in new information technologies based on textures, fractals, fractional operators and nonlinear dynamics methods created and developed by the author during 40 years are presented. The study is being conducted in the framework of the fundamental scientific area "Fractal Radio physics and Fractal Radio electronics: Designing Fractal Radio Systems", initiated and developed by the author in the V. A. Kotelnikov IRTE of the RAS from 1979 until the present day.

Keywords: radio physics; radiolocation; nonlinear dynamics; dimension theory; textures; fractals; scaling; fractional operators.

INTRODUCTION

The article discusses the main trends for the introduction of textures, fractals, fractional operators, non-Gaussian statistics and non-linear dynamics methods [1-10] into the fundamental problems of radio physics, radiolocation and a wide range of radio engineering to create new information technologies. The investigation is being conducted within the framework of the research area "Fractal Radio physics and Fractal Radio electronics: Designing Fractal Radio Systems", initiated and developed by the author in the V. A. Kotelnikov IRTE of the RAS from 1979 until the present day [11-28].

1. Main research areas

The main scientific areas developed by the author with students from 1979 until the present day can be classified as follows [11–28]:

1. Development of new information technologies for modern airborne and ground integrated radio engineering systems for remote sounding and monitoring of the environment, radiolocation, radio vision and navigation, operating in the ranges of optical, millimeter and centimeter waves (MMW and SHF band). Theoretical and experimental studies of the physical bases of scattering and propagation of radio waves, taking into account the spatially inhomogeneous characteristics of the medium being sounded.

2. Fundamental research in the area of textural and fractal approaches to the problems of radio physics, radio engineering, radiolocation, electrodynamics, electronics, control, and a wide range of related scientific and technical prospects. Empirical and theoretical modeling of the corresponding hereditary non-local real stochastic processes.

3. Application of correlation-extremal methods for solving problems of information search, detection, measurement of characteristics and tracking of dynamic fractal and non-fractal objects in stochastic images. Such tasks arise in radiolocation, natural resources survey, remote sounding, navigation, meteorology, information processing from unmanned aerial vehicles (UAVs) and synthetic aperture radars (SAR), medicine, biology, in the automation of scientific research, etc.

4. Development of the theory and experimental studies of broadband (BB) and ultrabroadband (UBB) signals and processes. Development of fractal and nonlinear BB and UBB signals, including fundamentally new types of signals (N-signals).

5. Development and elaboration of mathematical, including textural and fractal, methods for processing optical and radar images in information systems for various purposes (radiolocation, medicine, materials science, nanotechnology, scanning probe microscopes, astronomy, etc.).

6. Development and design of radio-electronic devices for the implementation of mathematical fractal methods for detecting super-weak multidimensional signals against the background of high-intensity non-Gaussian noise for new generation information systems.

7. Physics of the basic radiolocation equation for sounding fractal objects and randomly inhomogeneous media. Fractal-scaling or scale-invariant radiolocation, fractal multi-frequency MIMO-systems.

8. Theory of wave diffraction on a fractal multiscale surface. Multiple scattering of waves in fractal discrete randomly inhomogeneous media with relation to the radiolocation of self-similar multiple group targets. Waves in disordered large fractal systems (radiolocation, nanosystems, clusters of unmanned aerial vehicles and small spacecraft, space debris, etc.).

9. Application of the fractal theory in adaptive population methods for forming dynamic groups of UAVs with the organization of "distributed intelligence", the collective interaction of UAVs in a group and in the processing of incoming information in regards to the theory of their effective application. Development of solutions within the context of the concept of a distributed measuring environment, when each point of a certain dynamic environment is capable of performing sensory, measuring and informational functions, as well as based on a fractal-graph approach that makes for studying the growth of complex networks and the method for manipulating with such networks at the global level without a detailed description.

10. Formulation of the foundations of the fractal paradigm and the global fractal-scaling method. Elaboration and development of the functional principle "Maximum topology with minimum energy" for the received signal, that makes for more efficient use of the advantages of fractal-scaling processing of incoming information.

Fractal geometry is great and brilliant achievement of B. Mandelbrot (1924-2010). But its radiophysical / radiotechnical and practical implementation is the achievement of the world-famous Russian scientific school of fractal methods under the guidance of Prof. A.A. Potapov (V. A. Kotelnikov IRTE, RAS). In a metaphorical sense, it can be said that fractals represented a thin coating of amalgam on the powerful backbone of science at the end of the 20th century. Up to date, the attempts to diminish their significance and rely only on classical knowledge have suffered an intellectual fiasco. In December 2005 in the USA B. Mandelbrot (1924-2010) personally approved the developed classification of fractals, Fig.1. The numerous results obtained by the author on the above mentioned scientific areas have been concretized and illustrated in [18–20].

2. Theoretical foundations of the created fractal-scaling methods

In the fractal-scaling approach proposed and having been developed in the V.A. Kotelnikov IRTE of the RAS for 40 years, description and processing of signals and fields is carried out exclusively in fractional measure space using scaling hypotheses, heavy-tailed non-Gaussian stable distributions [1] and, as far as possible, using the apparatus of fractional integral derivatives [3-9, 11, 13, 14] . Note that if an equation includes a time fractional derivative, it is interpreted so as there is memory or, in the case of a stochastic process, non-Markovism.

The main property of fractals is the non-integral value of their dimension D . Development of the dimension theory started from the works of Poincare, Lebesgue, Brauer, Uryson and Menger. In various areas of mathematics, there occur sets that are negligible in one sense or another and are indistinguishable in terms of Lebesgue measure. To distinguish between such sets with a hugely complicated topological structure, it is necessary to involve nontraditional characteristics of

smallness, for example, capacity, potential, measures, and Hausdorff dimension, etc. The use of the Hausdorff fractional dimension, closely related to the concepts of entropy, fractals and strange attractors in the theory of dynamical systems, turned to be the most optimal [2, 4, 11, 13].



Fig.1. Meeting with B. Mandelbrot at his home in New York on 16.12.2005

The concept of a measure and Hausdorff dimension is defined by a p -dimensional measure with an arbitrary real positive number p introduced by Hausdorff in 1919. The concepts introduced by Hausdorff are based on the Carathéodory construct (1914). The Hausdorff dimension $\dim_H A$ is defined in terms of the Hausdorff α -measure of the set $mes_{H,\alpha}$ as

$$mes_{H,\alpha} = \liminf_{\varepsilon \rightarrow 0} \sum_{\Gamma(A)} [d(U)]^\alpha, \quad (1)$$

where the lower bound \inf is taken with respect to finite or counting coverings Γ of the set A by balls U , the diameters of which are $d(U) < \varepsilon$.

The dimension $\dim_H A$ is defined as such α_0 number that measure (1) is equal to zero for $\alpha > \alpha_0$, and for $\alpha < \alpha_0$ it is equal to infinity. In the general case, the concept of measure is not connected with either the metric or the topology. However, the Hausdorff measure can be developed in an arbitrary metric space based on its metric, and the Hausdorff dimension itself is connected with the topological dimension.

The basics of the modern theory of probability are the limit theorems on the convergence of distributions of sums of independent random variables to the so-called stable distributions: Gaussian or non-Gaussian. The former ones base on the central limit theorem, and the latter (non-Gaussian) ones base on the limit theorem proved by B.V. Gnedenko (1939) and V. Döblin (1940) [1]. In this case, the limit theorem imposes restrictions on the form of non-Gaussian distributions. In order for the distribution law $F(x)$ to belong to the domain of attraction of a stable law with a characteristic exponent α ($0 < \alpha < 2$), different from the Gaussian one, it is necessary and sufficient that

$$1) \quad \frac{F(-x)}{1-F(x)} \rightarrow \frac{c_1}{c_2} \quad \text{for } x \rightarrow \infty, \quad (2)$$

2) for each constant $k > 0$

$$\frac{1 - F(x) + F(-x)}{1 - F(kx) + F(-kx)} \rightarrow k^\alpha \quad \text{for } x \rightarrow \infty, \tag{3}$$

where the coefficients $c_1 \geq 0$, $c_2 \geq 0$, $c_1 + c_2 > 0$, $0 < \alpha < 2$.

To prove (2) and (3) it is necessary and sufficient that with a certain selection of B_n constants, the following conditions were met [1, p. 189]:

$$nF(B_n x) \rightarrow \frac{c_1}{|x|^\alpha} \quad (x < 0),$$

$$n[1 - F(B_n x)] \rightarrow \frac{c_2}{x^\alpha} \quad (x > 0), \tag{4}$$

$$\lim_{\varepsilon \rightarrow 0} \overline{\lim}_{n \rightarrow \infty} n \left\{ \int_{|x| < \varepsilon} x^2 dF(B_n x) - \left[\int_{|x| < \varepsilon} x dF(B_n x) \right]^2 \right\} = 0.$$

The smaller the α value, the longer the distribution tail and the more it differs from the Gaussian distribution. For $1 < \alpha < 2$, stable laws have a mathematical expectation; for $0 < \alpha \leq 1$, stable laws have neither dispersions nor mathematical expectations. Conditions (2)-(4) determine the so-called *non-Gaussian statistics*.

In ordinary statistics, fluctuations tend to zero when the sample size or the number of N terms increases. This guarantees the asymptotically exact repeatability of averages and is the source of the traditional successes of classical statistical methods in radiolocation. For Levy statistics, the situation may differ radically. With an increase in the sample size, the accuracy of statistical estimations does not improve! The standard form of the central limit theorem predicts disappearing fluctuations for large N , and from the generalized central limit theorem (for $\alpha < 1$) it follows that the fluctuations are significant for arbitrarily large N . At the same time, for $\alpha < 1$, a case of global non-ergodicity of processes is observed.

Note one more fact. Non-integral values of the α index in the range of $1 < \alpha \leq 2$ correspond to the generalized Brownian motion with long-term correlations and statistical self-similarity, i.e. fractal process. Self-similarity is mathematically expressed by power laws. The fractal dimension of the probability space of the time series is equal to α index:

$$\alpha = 1/H. \tag{5}$$

where H is the Hurst exponent. It is necessary to distinguish the “ordinary” fractal dimension D of the signal or image under study and the fractal dimension determined by the α index. If D characterizes the “curvedness” of objects, then α characterizes the tail thickness of probability distributions [4, 11, 13].

In V.A. Kotelnikov IRTE of the RAS, various original methods for measuring the fractal dimension D have been developed; including the dispersion method, the method taking into account singularities, functionals, triad, based on the Hausdorff metric, sample subtraction, based on the operation "Exclusive OR", etc. [11, 13, 16]. The local dispersion method for measuring the fractal dimension D is based on measuring the dispersion of the intensity / brightness σ_i^2 of optical or radar image fragments by two spatial scales δ_i^2 :

$$D \approx \frac{\ln \sigma_2^2 - \ln \sigma_1^2}{\ln \delta_2 - \ln \delta_1}, \quad i = 1 \text{ or } 2. \tag{6}$$

In the Gaussian case, the dispersive dimension of a random sequence converges to the Hausdorff dimension of the corresponding stochastic process. The principal difficulty is that any numerical method involves discretization (or discrete approximation) of the process or object being analyzed; and discretization destroys fractal properties. To resolve this conflict, it is necessary to develop a special theory based on the methods of fractal interpolation and approximation.

The fractal dimension D or its signature $D(t, f, F)$ in different parts of the surface image is a texture measure. Fractal methods can function at all signal levels: amplitude, frequency, phase and polarization. Fractional mathematical analysis has a long history and extremely rich content [5, 14]. Ideas about fractional integro-differentiation interested many prominent scientists: Leibniz, Euler, Liouville, and others. Interest in fractional mathematical analysis arose almost simultaneously with the origin of classical analysis (as early as in 1695 G. Leibniz mentioned this fact in letters to G. Lopital when considering differentials and derivatives of $\frac{1}{2}$ order). Note the set of papers by the associate member of the Petersburg Academy of Sciences (1884) A.V. Letnikov, who, during his 20 years of scientific work, developed a complete theory of differentiation with an arbitrary index [14].

At present, the expression for the fractional derivative D_{at}^α in the form proposed by Riemann and Liouville (${}_{RL}D_{at}^\alpha$) is most frequently used.

The operator of integro-differentiation in the sense of Riemann-Liouville of the fractional order $\alpha \in R$ originated at the point a is defined as follows [3-7, 9, 11, 13, 14]:

$${}_{RL}D_{at}^\alpha f(t) = \frac{\text{sign}(t-a)}{\Gamma(-\alpha)} \int_a^t \frac{f(\tau)}{|t-\tau|^{\alpha+1}} d\tau, \quad \alpha < 0, \tag{7}$$

$${}_{RL}D_{at}^\alpha = f(t), \quad \alpha = 0, \tag{8}$$

$${}_{RL}D_{at}^\alpha = \text{sign}^n(t-a) \frac{d^n}{dt^n} D_{at}^{\alpha-n} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t (t-\tau)^{n-\alpha-1} f(\tau) d\tau, \tag{9}$$

where $n-1 < \alpha \leq n$, $n \in N$; $\text{sign}(z)$ is determined by the equalities $\text{sign} 0 = 0$, $\text{sign} z = z/|z|$, ($z \neq 0$); $\Gamma(\alpha)$ is a gamma function.

For functions differentiable on the interval $[a, b]$, the definitions of fractional derivatives according to Riemann-Liouville and Letnikov are equivalent. Currently, the Caputo formula [6, 7, 14] is widely used:

$${}_CD_{at}^\alpha f(t) = \text{sign}^n(t-a) {}_{RL}D_{at}^{\alpha-n} f^{(n)}(t), \quad n-1 < \alpha \leq n, \quad n \in N. \tag{10}$$

The Riemann-Liouville and Caputo derivatives are associated by the formula [11]

$${}_CD_{at}^\alpha f(t) = {}_{RL}D_{at}^\alpha f(t) - \sum_{k=0}^{n-1} \frac{f^{(k)}(\tau)}{\Gamma(k-\alpha+1)} |\tau-t|^{k-\alpha}, \quad n-1 < \alpha \leq n, \quad n \in N. \tag{11}$$

In the case $\alpha = n$ we get

$${}_{RL}D_{at}^n f(t) = {}_CD_{at}^n f(t) = \text{sign}^n(t-a) \frac{d^n}{dt^n} f(t), \quad n \in N. \tag{12}$$

The Caputo derivative has the same physical interpretation as the Riemann-Liouville derivative. In particular, for $f(0) = 0$ and $0 < \alpha < 1$ there is the exact equality

$${}_CD_{0t}^\alpha f(t) = {}_{RL}D_{0t}^\alpha f(t). \tag{13}$$

When comparing these derivatives, pay attention to the fact that in order to compute the Riemann-Liouville derivative it is necessary to know the function values, and as for the Caputo derivative, one should know the derivative values, which is much more complicated. Some advantage of the Caputo derivative is that it is zero for a constant function, which is more usual for a researcher.

Conclusion

For the first time, the problem in the title of the work began to be studied by the author exactly 40 years ago at the IRE of the Academy of Sciences of the USSR when carrying out a fundamental research cycle related to the development of new breakthrough radio physical technologies for radiolocation. The main objective was to detect various low-contrast objects against the background of heavy clutter from the ground surface on the base of one-dimensional (signal) and multi-dimensional (optical and radar images) sample.

First, the *complete* families of textural features were studied (for the first time ever), then the transition to fractal features started (again first-ever). Later on, the author united these families of features in a common cluster of features. Huge data arrays obtained by the author in optics and on millimeter waves in long-term joint field experiments with leading enterprises of the USSR served as the source material. Up to the beginning of 2019, the author's priority in the above-mentioned scientific fields is confirmed by more than 1,000 scientific works and 37 domestic and foreign monographs and individual chapters in them in Russian and English [28].

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