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RESISTANCE MOMENT AT ROTATION OF AN ELLIPSOID IN VISCOUS FLUID

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The slow rotation of a solid body in a liquid, which is affected by the resistance force, is considered on the example of an ellipsoid. This force is created by both friction forces and pressure forces resulting from the effect of the attached mass. A method of resistance calculation for an arbitrary ellipsoidal body is proposed. Analytical formulas for the resistance forces of an ellipsoid rotating in a viscous fluid are established. The results obtained are applicable for the calculation of multiphase flows in technical devices.

Keywords: ellipsoid, resistance force, rotation, unsteady motion, viscosity

Introduction

The movement of multiphase media in various technological processes is of great interest. These include the task of effluent treatment, sedimentation of dispersed mixtures, the deposition of combustion products emitted from the pipes of industrial enterprises and so on. In nature, we also often observe multiphase currents, in particular, the movement of dust and rain clouds, vortex tornadoes and many other things. Prediction of the evolution of such dispersed two-phase systems should be based on known calculation and experimental data on the strength of the interaction of a single particle of a given shape with a carrier medium taking into account all the properties of the medium and the laws of particle motion.

The solution of the complete problem, taking into account the interaction of each particle with the carrier medium, is an insoluble problem due to a large number of moving particles. As a rule, they are limited by the phenomenological law of the hydrodynamic resistance of a single particle in a carrier medium. The assumption of spherical particle shape is the most common and quite acceptable. In cases, when the shape of the particle differs from the spherical shape, a certain effective radius of a conforming spherical particle is determined, which, when moving under the same conditions, will experience the same resistance of the medium as the original particle.

This approach reasonably led to the fact that most of the efforts to determine the law of resistance were devoted to the study of spherical particles. In many cases, however, a moving particle can more accurately be described as an ellipsoid. And in this regard, studies devoted to the flow around ellipsoids, in particular, ellipsoids of rotation (spheroids) are of interest. In [1, 2], the results of analytical computation on the flow parallel to the main axis of spheroids at small Reynolds numbers are presented. In [3], the results of a numerical calculation of the flow around a uniform stationary stream of spheroids of various shapes with Re <100 in the axially symmetrical formulation are presented, and the values of the integral characteristics are given. In [4–7], the calculated data on the flow around spheroids at Reynolds numbers of about 100 are shown. The values of the drag coefficient depending on the degree of elongation are presented.

The results of numerical and experimental studies make it possible to build correlation dependences used in determining the resistance forces during the motion of particles. Unfortunately, all expressions are approximate and have limits of applicability. From our point of view, attempts to clarify the law of hydrodynamic resistance of a moving body in various conditions will continue for a very long period of time.

The information on the law of resistance of an arbitrary ellipsoid, which makes it possible to more accurately describe the shape of a body moving in a viscous medium, remains relevant. In [8, 9], the problem of slow stationary flow of a viscous flow around a triaxial ellipsoid is solved, a simple calculation formula for its resistance is indicated. In the case of unsteady flows (rotation of the body, acceleration of the flow), the body is affected by the resistance force created by both the friction forces and the pressure forces resulting from the effect of the added mass. Therefore, a more accurate description of the shape of the body becomes more important.

In this paper, the authors consider the actual problem of determining the resistance force during the slow rotation of an ellipsoid in a viscous fluid based on the application of a tension-compression transformation.

1. Determination of the flow fields of a rotating sphere and an ellipsoid.

In exceptional cases, the system of equations of hydromechanics

$$\mathrm{Sh}\frac{\partial \vec{v}}{\partial t} + \nabla \frac{1}{2}\vec{v}\vec{v} - \vec{v}\times \mathrm{rot}\,\vec{v} = -\mathrm{Eu}\,\nabla p - \frac{1}{\mathrm{Re}}\mathrm{rot}\,\mathrm{rot}\,\vec{v}, \quad \mathrm{div}\,\vec{v} = 0$$

breaks down into a system for velocity

$$\mathrm{div}\,\vec{v} = 0, \qquad \mathrm{rot}\,\mathrm{rot}\,\vec{v} = 0 \tag{1}$$

and equations for finding the pressure field

$$\mathrm{Sh}\frac{\partial \vec{v}}{\partial t} + \nabla \frac{1}{2}\vec{v}\vec{v} - \vec{v}\times \mathrm{rot}\,\vec{v} = -\mathrm{Eu}\,\nabla p, \tag{2}$$

here, equations (1) and (2) use the generally accepted notations for the operators of differentiation, velocity, pressure, and Strouhal, Euler, and Reynolds numbers.

One of these cases is the case of slow rotation of a sphere in a viscous incompressible fluid, when due to sticking the sphere initiates movement in the surrounding unlimited medium, as a result of which a velocity field is established in the fluid [10]

$$\vec{v} = \sigma^{-3}\vec{\omega}\times\vec{\sigma} = \mathrm{rot}\frac{\vec{\omega}}{\sigma}, \qquad (\vec{\sigma} = \xi\,\vec{i} + \eta\,\vec{j} + \varsigma\,\vec{k}, \qquad \vec{\omega} = \omega_1\,\vec{i} + \omega_2\,\vec{j} + \omega_3\,\vec{k}\ ) \tag{3}$$

$$|\vec{\gamma}| = |\vec{\delta}| = |\vec{\rho}| = 1 \quad \omega_1^2 + \omega_2^2 + \omega_3^2 = 1$$

satisfying the subsystem (1) and the boundary conditions on the sphere itself ($\sigma = 1$)

$$\vec{v}(1) = \vec{\omega}\times\vec{\sigma} \tag{4}$$

and away it (at $\sigma \to \infty$)

$$\vec{v}(\infty) = 0,$$

where $\vec{\sigma}$ is the radius of the vector, $\vec{\omega}$ is the angular velocity of rotation.

In this case, the friction force resistance moment acts on the rotating sphere

$$\vec{M}(\gamma) = 2\int \vec{\sigma}\times\gamma\, d\vec{S} = -8\pi\vec{\omega},$$

or in dimensional values [10]

$$\vec{M}(\gamma) = -8\pi\mu R^3\vec{\omega}, \tag{5}$$

here γ is the strain velocity tensor, μ is the medium viscosity, R is the sphere radius.

As for equation (2), on the basis of which the pressure p is found, and the general formula

$$\vec{M}(p) = \int p\,\vec{\sigma}\times d\vec{S} \tag{6}$$

for the moment of pressure forces, they can effectively be used for stationary flows. But rotational motions (or their superposition, which is allowed by the system (1)) are by their nature non-stationary. Therefore, to take into account nonstationarity, it is assumed that the moment $\vec{M}(p)$ can be calculated based on the integral theorem on the change of angular momentum

.

$$\frac{d}{dt}\int \vec{\sigma}\times\vec{v}\,dm = -\int p\,\vec{\sigma}\times\, d\vec{S},$$

where dm is the element of mass, $d\vec{S}$ is the element of the surface, on the assumption that the liquid mass rotates like a solid body. Then on the side of liquid the resistance moment will act on the solid.

$$\vec{M}(p) = -\frac{d}{dt}J_g\vec{\omega} = -J_g\frac{\partial\vec{\omega}}{\partial t} - \vec{\omega}\times J_g\vec{\omega},$$

where J_g is the inertia tensor of the solidified body with a uniform density equal to the liquid density. In the case of a sphere, the vector product $\vec{\omega}\times J_g\vec{\omega}$ (it is the one that is obtained by using (6) and the Bernoulli integral for a steady flow) disappears by virtue of central symmetry.

Thus, it can be concluded that when removing active moments supporting the rotation of the sphere near its center of mass, the further (damped) motion of the sphere will be determined by the equation

$$(J_b + J_g)\frac{\partial\vec{\omega}}{\partial t} = -8\pi\mu R^3\vec{\omega}, \tag{7}$$

in which, along with the tensor $\boldsymbol{J_g}$ of the hardened liquid sphere, there occurs inertia tensor $\boldsymbol{J_b}$ of the solid sphere or the hollow one itself.

Equation (7) shows that with the active effects being removed, the previous rotation of a solid sphere at a velocity $\vec{\boldsymbol{\omega}}_0$ decreases exponentially

$$\vec{\omega} = \vec{\omega}_0\exp\left(-\frac{15\mu t}{(\rho_b+\rho_g)R^2}\right). \tag{7′}$$

Consider the flow generated by a rotating solid ellipsoid

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1 \tag{8}$$

at an angular velocity

$$\vec{\omega} = \omega_1\,\vec{i} + \omega_2\,\vec{j} + \omega_3\,\vec{k} \tag{9}$$

around some axis passing through its center ($\vec{\boldsymbol{i}}, \vec{\boldsymbol{j}}, \vec{\boldsymbol{k}}$ fixed along the main central axes of the body inertia, and the directions of the semi-axes $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ correspond to them).

It is advisable in (8) to replace the ratio of dimensional quantities of $\boldsymbol{x/a}$ type with the relations of dimensionless quantities. This can be done by dividing the numerators and denominators by the same linear scale $\boldsymbol{L}$ used in nondimensionalization of the Navier-Stokes equations. In this case, we obtain the canonical equation of the ellipsoid as follows

$$\left(\frac{x}{\lambda_1}\right)^2 + \left(\frac{y}{\lambda_2}\right)^2 + \left(\frac{z}{\lambda_3}\right)^2 = 1.$$

Then the transformation in vector form

$$\xi\,\vec{i} + \eta\,\vec{j} + \varsigma\,\vec{k} = \vec{\sigma} = \left(\frac{x}{\lambda_1}\right)\vec{i} + \left(\frac{y}{\lambda_2}\right)\vec{j} + \left(\frac{z}{\lambda_3}\right)\vec{k}$$

$$\xi^2 + \eta^2 + \varsigma^2 = \sigma^2 = \left(\frac{x}{\lambda_1}\right)^2 + \left(\frac{y}{\lambda_2}\right)^2 + \left(\frac{z}{\lambda_3}\right)^2$$

will translate the sphere of unit radius into a preset ellipsoid according to the law of simple tension-and-compression deformation [9]. In this case, the velocity field (3) will also take place in case of a rotating ellipsoid, and it satisfies in variables x, y, z, both equations (1) and boundary conditions (4), (5). Moreover, each term from (3), i.e. any velocity satisfies the above requirements,

$$\vec{v}_1 = \sigma^{-3}\omega_1 \vec{i}\times\vec{\sigma},\ \ \vec{v}_2 = \sigma^{-3}\omega_2 \vec{j}\times\vec{\sigma},\ \vec{v}_3 = \sigma^{-3}\omega_3 \vec{k}\times\vec{\sigma},$$

that is why, the use of expression (9) is quite justifiable, since in fact, it follows the principle of superposition of the simplest flows. But at the same time, the principle of superposition applies to pressure or the squared velocity in the form

$$v^2 = (\vec{v}_1+\vec{v}_2+\vec{v}_3)^2 = \sigma^{-6}(\omega_1{}^2(\eta^2+\varsigma^2)+\omega_2{}^2(\xi^2+\varsigma^2)+\omega_3{}^2(\xi^2+\eta^2)-2\omega_1\omega_2\xi\eta-2\omega_2\omega_3\eta\varsigma-2\omega_1\omega_3\xi\varsigma)$$

2. Calculation of the projections of the principal moment of resistance from friction forces

First turning attention to calculation of the principal moment generated by friction forces, it is necessary to find the values of the components γ_{ij} of the strain velocity tensor. Since, based on (3), the values of the generalized rates at any point are represented by the equalities

$$\dot{x} = \lambda_1\sigma^{-3}(\omega_2\lambda_3{}^{-1}z - \omega_3\lambda_2{}^{-1}y),$$

$$y = \lambda_2\sigma^{-3}(\omega_3\lambda_1{}^{-1}x - \omega_1\lambda_3{}^{-1}z),$$

$$\dot{z} = \lambda_3\sigma^{-3}(\omega_1\lambda_2{}^{-1}y - \omega_2\lambda_1{}^{-1}x),$$

then using them and the general formula,

$$2\gamma_{ij} = \left(\left|\frac{\partial\vec{\sigma}}{\partial q_i}\right|\left|\frac{\partial\vec{\sigma}}{\partial q_j}\right|\right)^{-1}\left[\frac{d}{dt}\left(\frac{\partial\vec{\sigma}}{\partial q_i}\frac{\partial\vec{\sigma}}{\partial q_j}\right)+\frac{\partial\vec{\sigma}}{\partial q_k}\left(\frac{\partial\vec{\sigma}}{\partial q_i}\frac{\partial\dot{q}_k}{\partial q_j}+\frac{\partial\vec{\sigma}}{\partial q_j}\frac{\partial\dot{q}_k}{\partial q_i}\right)\right],$$

it can be found that on the surface of the ellipsoid the components of the tensor take the values

$$\gamma_{xx} = -3\xi(\omega_2\varsigma - \omega_3\eta),\ \gamma_{yy} = -3\eta(\omega_3\xi - \omega_1\varsigma),\ \gamma_{zz} = -3\varsigma(\omega_1\eta - \omega_2\xi)$$

$$\gamma_{xy} = -\frac{3}{2}(-\omega_1\xi\varsigma + \omega_2\eta\varsigma + \omega_3(\xi^2-\eta^2)),\ \gamma_{yz} = -\frac{3}{2}(\omega_1(\eta^2-\varsigma^2) - \omega_2\xi\eta + \omega_3\xi\varsigma))$$

$$\gamma_{xz} = -\frac{3}{2}(\omega_1\xi\eta + \omega_2(\varsigma^2-\xi^2) - \omega_3\varsigma\eta)).$$

Hereinafter, to abbreviate expressions, the following notations are used

$$\xi = \cos\theta,\ \eta = \sin\theta\cos\varphi,\ \varsigma = \sin\theta\sin\varphi;\ 0\le\theta\le\pi,\ 0\le\varphi\le 2\pi.$$

In these notations, the radius-vector of the current point of the body surface is taken as

$$\vec{\sigma}_s = \vec{i}\lambda_1\xi + \vec{j}\lambda_2\eta + \vec{k}\lambda_3\varsigma,$$

so that for a directed element of the ellipsoid surface, the following is correct

$$d\vec{S} = (\vec{i}\lambda_2\lambda_3\xi + \vec{j}\lambda_1\lambda_3\eta + \vec{k}\lambda_1\lambda_2\varsigma)\sin\theta\, d\theta\, d\varphi. \qquad (10)$$

After performing calculations recommended by the formula

$$\vec{M}(\gamma) = 2\int\vec{\sigma}_s\times\gamma\, d\vec{S} = 2\int\vec{\sigma}_s\times(\lambda_2\lambda_3\xi\vec{\gamma}_x + \lambda_1\lambda_3\eta\vec{\gamma}_y + \lambda_1\lambda_2\varsigma\vec{\gamma}_z)\sin\theta\, d\theta\, d\varphi$$

it turns out that the projections of the principal friction moment on the principal central inertia axes are determined by the equalities

$$M_a(\gamma) = -\frac{4}{5}\pi(\lambda_2+\lambda_3)(\lambda_2\lambda_3 + 2\lambda_1(\lambda_2+\lambda_3))\,\omega_1,$$

$$M_b(\gamma) = -\frac{4}{5}\pi(\lambda_1+\lambda_3)(\lambda_1\lambda_3 + 2\lambda_2(\lambda_1+\lambda_3))\,\omega_2 \qquad (11)$$

$$M_c(\gamma) = -\frac{4}{5}\pi(\lambda_1+\lambda_2)(\lambda_1\lambda_2 + 2\lambda_3(\lambda_1+\lambda_2))\,\omega_3$$

.

Translation of the results (11) into the calculated dimensional form is carried out by multiplying them by the complex

$$\mu \frac{V}{L} L^3 = \mu L^3 T^{-1},$$

then it turns into

$$M_a(\gamma) = -\frac{4}{5}\pi\mu\omega_1 (b+c)(bc+2a(b+c)),$$

$$M_b(\gamma) = -\frac{4}{5}\pi\mu\omega_2 (a+c)(ac+2b(a+c)), \qquad (12)$$

$$M_c(\gamma) = -\frac{4}{5}\pi\mu\omega_3 (a+b)(ab+2c(a+b)),$$

that automatically, at $\boldsymbol{a = b = c = R}$, brings to the previously cited result (5) for a sphere. Formulas (12) are convenient because they can be applied to plates (round disks); for needle bodies they lead to a zero result.

3. Calculation of the projections of the principal resistance moment from pressure forces

Starting to determine the resistance moment created by pressure forces , one can first stop on the calculation of the part that arises from the interaction of rotations. In this regard, it is taken into account that

$$\vec{M}(p) = -\int p\vec{\sigma} \times d\vec{S} \approx \frac{1}{2}\int v_s{}^2 \vec{\sigma}_s \times d\vec{S}$$

where $\boldsymbol{d\vec{S}}$ is defined by (10), and

$$v_s^2 = \left|\vec{\omega} \times \vec{\sigma}_s\right|^2, \; \vec{\sigma}_s = \lambda_1 \xi \vec{i} + \lambda_2 \eta \vec{j} + \lambda_3 \varsigma \vec{k}.$$

After performing the calculations, it is found that approximate dimensionless expressions take place for the projections of the moment

$$M_a(p) = -\frac{4}{15}\pi\lambda_1\lambda_2\lambda_3(\lambda_2^2 - \lambda_3^2)\omega_2\omega_3,$$

$$M_b(p) = -\frac{4}{15}\pi\lambda_1\lambda_2\lambda_3(\lambda_3^2 - \lambda_1^2)\omega_1\omega_3,$$

$$M_c(p) = -\frac{4}{15}\pi\lambda_1\lambda_2\lambda_3(\lambda_1^2 - \lambda_2^2)\omega_1\omega_2,$$

that after multiplying by the complex

$$\rho V^2 L^3 = \rho T^{-2} L^5$$

take a dimensional form

$$M_a(p) = -\frac{4}{15}\pi\rho\, a\, b\, c\,(b^2 - c^2)\omega_2\omega_3,$$

$$M_b(p) = -\frac{4}{15}\pi\rho abc\,(c^2 - a^2)\omega_1\omega_3, \qquad (13)$$

$$M_c(p) = -\frac{4}{15}\pi\rho\, a\, b\, c\,(a^2 - b^2)\omega_1\omega_2.$$

From (13) it can be seen that the resistance moment from pressure forces while ignoring angular acceleration occurs only as a result of the interaction of rotational motions, that is, in those cases when the rotation axis does not coincide with any principal central inertia axes of the ellipsoid itself. Expressions (13) do not apply to any other bodies, as they are only drawn up for an ellipsoid.

In order to be applied to other bodies, it is necessary (13) to represent a more general form by introducing into consideration the inertia tensor of the ellipsoid

$$J=\frac{1}{5}m\begin{pmatrix} b^2+c^2 & 0 & 0 \\ 0 & a^2+c^2 & 0 \\ 0 & 0 & a^2+b^2 \end{pmatrix},\; m=\frac{4}{3}\pi\rho abc\ .$$

In this case (13) take the form

$$\vec{M}(p)=-\vec{\omega}\times J\vec{\omega}$$

and this expression immediately informs that in the general case the resistance moment should be represented as

$$\vec{M}(p)=-\frac{d}{dt}J\vec{\omega}=-J\frac{\partial\vec{\omega}}{\partial t}-\vec{\omega}\times J\vec{\omega}, \tag{14}$$

as stated by the integral theorem on the change in the moment of momentum

$$\frac{d}{dt}\int\vec{\sigma}\times\vec{v}\,dm=-\int\rho\vec{\sigma}\times d\vec{S}\,.$$

Now the formula (14) makes for obtaining information on the arising resistance moment from pressure forces for other bodies. To do this, it suffices in (14) to choose the inertia tensor ***J*** corresponding to the body chosen. For example, for a plate ($a=0$) you should use

$$J=\frac{1}{4}m\begin{pmatrix} b^2+c^2 & 0 & 0 \\ 0 & c^2 & 0 \\ 0 & 0 & b^2 \end{pmatrix}.$$

4. Determination of the damped motion of the bodies of some configurations in a viscous medium.

The obtained information for the resistance moments in the form of (13), (14) makes for conclusion that after removing the active effects on maintaining rotation, the subsequent motion of a rigid body near its inertia center in a viscous medium can be described by the equations

1) for a uniform ellipsoid:

$$\begin{aligned}
&(m_b+m_g)\left[(b^2+c^2)\dot{\omega}_1+(b^2-c^2)\omega_2\omega_3\right]+4\pi\mu\omega_1(b+c)(bc+2a(b+c))=0,\\
&(m_b+m_g)\left[(a^2+c^2)\dot{\omega}_2+(c^2-a^2)\omega_1\omega_3\right]+4\pi\mu\omega_2(a+c)(ac+2b(a+c))=0,\\
&(m_b+m_g)\left[(a^2+b^2)\dot{\omega}_3+(a^2-b^2)\omega_1\omega_2\right]+4\pi\mu\omega_3(a+b)(ab+2c(a+b))=0\,;
\end{aligned} \tag{15}$$

2) for a homogeneous plate $a=0$:

$$\begin{aligned}
&(m_b+m_g)\left[(b^2+c^2)\dot{\omega}_1+(b^2-c^2)\omega_2\omega_3\right]+\frac{16}{5}\pi\mu\omega_1(b+c)bc=0,\\
&(m_b+m_g)\left[\dot{\omega}_2+\omega_1\omega_3\right]+\frac{32}{5}\pi\mu\omega_2 b=0\\
&(m_b+m_g)\left[\dot{\omega}_3-\omega_1\omega_2\right]+\frac{32}{5}\pi\mu\omega_3 c=0\,;
\end{aligned} \tag{16}$$

The corresponding equations for the motion of a rigid body in vacuum are obtained from those presented for $m_g=0$ and $\mu=0$. They conform with the classical ones [11].

For the sphere ($a=b=c$), equations (15) turn into:

$$\dot{\omega}_i+20\pi\frac{c\mu}{(m_b+m_g)}\omega_i=0,\; i=1,2,3$$

the solution is:

.

$$\omega_i = \omega_{i0}\exp(-k_1 t),\; i = 1,2,3,\; k_1 = 20\pi\frac{c\mu}{(m_b + m_g)}.$$

For the motion of an ellipsoid with axial symmetry ($\boldsymbol{b} = \boldsymbol{c}$), equations (15) become

$$\dot{\omega}_1 + \frac{4\pi(4a + c)\mu\omega_1}{(m_b + m_g)} = 0,$$

$$\dot{\omega}_2 - \frac{a^2 - c^2}{a^2 + c^2}\omega_1\omega_3 + 4\pi\frac{(a + c)(3a + 2c)}{a^2 + c^2}\frac{c\mu\omega_2}{(m_b + m_g)} = 0,$$

$$\dot{\omega}_3 + \frac{a^2 - c^2}{a^2 + c^2}\omega_1\omega_2 + 4\pi\frac{(a + c)(3a + 2c)}{a^2 + c^2}\frac{c\mu\omega_3}{(m_b + m_g)} = 0$$

and integrate analytically in elementary functions.

From the first follows

$$\omega_1 = \omega_{10}\exp(-k_2 t),\; k_2 = 4\pi\frac{(4a + c)\mu}{m_b + m_g},$$

and the second two can be written as equations

$$\omega_2^2\frac{d}{dt}\frac{\omega_3}{\omega_2} + \frac{a^2 - c^2}{a^2 + c^2}\omega_1\omega_2^2\left(1 + \left(\frac{\omega_3}{\omega_2}\right)^2\right) = 0,$$

$$\frac{d}{dt}(\omega_2^2 + \omega_3^2) + 4\pi\frac{(a + c)(3a + 2c)}{a^2 + c^2}\frac{2c\mu}{(m_b + m_g)}(\omega_2^2 + \omega_3^2) = 0.$$

Using them the following are found

$$\frac{\omega_3}{\omega_2} = \mathrm{tg}\left[\mathrm{arctg}\frac{\omega_{30}}{\omega_{20}} - \frac{a^2 - c^2}{a^2 + c^2}\int_0^t \omega_1 dt\right] = \mathrm{tg}\Phi,$$

$$\Phi = \mathrm{arctg}\frac{\omega_{30}}{\omega_{20}} - \frac{a^2 - c^2}{a^2 + c^2}\int_0^t \omega_1 dt,$$

$$\omega_2^2 + \omega_3^2 = (\omega_{20}^2 + \omega_{30}^2)\exp\left(-2\frac{c(a + c)(3a + 2c)}{(4a + c)(a^2 + c^2)}k_2 t\right).$$

That is

$$\omega_2^2 = (\omega_{20}^2 + \omega_{30}^2)\exp\left(-2\frac{c(a + c)(3a + 2c)}{(4a + c)(a^2 + c^2)}k_2 t\right)\cos^2\Phi,$$

$$\omega_3^2 = (\omega_{20}^2 + \omega_{30}^2)\exp\left(-2\frac{c(a + c)(3a + 2c)}{(4a + c)(a^2 + c^2)}k_2 t\right)\sin^2\Phi$$

The resulting system of equations for a rotating circular disk $a = 0$, $b = c$ in a resisting medium is integrated in a similar way.

$$\dot{\omega}_1 + \frac{16}{5}\pi\frac{\mu c\omega_1}{m_b + m_g} = 0,$$

$$\dot{\omega}_2 + \omega_1\omega_3 + \frac{32}{5}\pi\frac{\mu c\omega_2}{m_b + m_g} = 0,$$

$$\dot{\omega}_3 - \omega_1\omega_2 + \frac{32}{5}\pi\frac{\mu c\omega_3}{m_b + m_g} = 0.$$

From it for rotation rates follow the basic and explaining formulas

$$\omega_1 = \omega_{10} \exp(-k_3 t)\,,\ \Phi_1 = \operatorname{arctg}\frac{\omega_{20}}{\omega_{30}} - \int_0^t \omega_1 dt\,,\ k_3 = 16\pi \frac{c\mu}{(m_b + m_g)}$$
$$\omega_2^2 = (\omega_{20}^2 + \omega_{30}^2)\exp(-2k_3 t)\sin^2\Phi_1\,,$$
$$\omega_3^2 = (\omega_{20}^2 + \omega_{30}^2)\exp(-2k_3 t)\cos^2\Phi_1\,.$$

The kinetic energy during the rotational motion of a solid body in accordance with the definition can be found by the formula

$$E = \frac{1}{2}(J_{xx}\omega_1^2 + J_{yy}\omega_2^2 + J_{zz}\omega_3^2)\,.$$

The dissipation of energy with time is determined by the ratio, which is equal for the sphere:

$$R1(t) = \frac{E}{E_0} = \exp(-2k_1 t)\,,$$

for a round disk of c radius:

$$R3(t) = \frac{E}{E_0} = \frac{2c^2\omega_1^2 + c^2(\omega_2^2 + \omega_3^2)}{2c^2\omega_{10}^2 + c^2(\omega_{20}^2 + \omega_{30}^2)} = \exp(-2k_3 t)\,.$$

In the studied motion of the rotation ellipsoid ($b = c$) by inertia in a resisting medium there should be

$$E = \frac{1}{10}(m_b + m_g)(2c^2\omega_1^2 + (a^2 + c^2)(\omega_2^2 + \omega_3^2))$$

and, therefore, at $R2 = \dfrac{E}{E_0}$ there will be

$$R2(t) = \frac{2c^2\omega_{10}^2\exp(-2k_2 t) + (a^2 + c^2)(\omega_{20}^2 + \omega_{30}^2)\exp\left(-2\dfrac{(a+c)(3a+2c)}{(4a+c)(a^2+c^2)}k_2 t\right)}{2c^2\omega_{10}^2 + (a^2 + c^2)(\omega_{20}^2 + \omega_{30}^2)}\,.$$

Figure 1 shows the dependencies of $R2(t)$ in case of different ratios.

$a/c = 1, 5, 10$, $k_2 = 1$ и $\omega_{10} = \omega_{20} = \omega_{30} = \mu/c^2\rho_b = 1$.

The graphs are illustrative and only qualitatively show the rapid decay of the rotational motion.

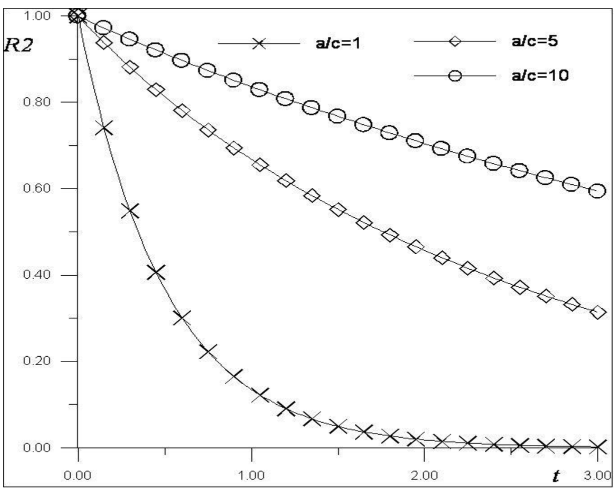

Fig.1. Reduction of the relative kinetic energy of rotation

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To determine the rotation law of a solid body in a resisting medium, it is necessary to add the Euler kinematic relations to the equations obtained [11]

$$\dot{\psi}\sin\theta\sin\varphi+\dot{\theta}\cos\varphi=\omega_1,$$

$$\dot{\psi}\sin\theta\cos\varphi-\dot{\theta}\sin\varphi=\omega_2,$$

$$\dot{\psi}\cos\theta+\dot{\varphi}=\omega_3,$$

in which ψ, φ, θ indicate Euler angles, but this question becomes of interest only if there are active forces.

Conclusion

Thus, by the example of an ellipsoid, a slow rotation of a solid body in a liquid is considered, taking into account the fact that it is exerted by the resistance force created by both the friction forces and the pressure forces resulting from the effect of the added mass.

Derived equations of rotation by inertia in a resisting medium near the geometric center for ellipsoid bodies can be applied to plates into which an ellipsoid can degenerate. The obtained expressions can be easily applied in the calculations of multiphase flows in technical devices and in the study of natural phenomena.

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