UDC 519.63; 533.697.2; 533.6.011.5

NUMERICAL SOLUTION OF THE PROBLEM OF SUPERSONIC GAS FLOW IN TWO-DIMENSIONAL CHANNEL WITH THE OSCILLATING UPPER WALL

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In paper we solve the problem of supersonic gas flow in two-dimensional channel with the moving upper wall making oscillations according to the harmonic law. In order to get a numerical solution for gas dynamics equations we have implemented the difference scheme with space and time approximation of the first order using Van Leer's method to compute fluxes. A special form of fluxes in the gas dynamics equations is given, which enables to calculate fluxes on cell faces of difference mesh using Van Leer's method. Depending on a type of harmonic law and initial gas inflow conditions, the peculiarities of angle-shock wave propagation in moving curvilinear domains have been investigated. It has been determined that under a particular oscillation frequency the presence of wall oscillation practically doesn't have an effect on the flow regime inside the domain. While comparing the numerical solution obtained due to our program with the one obtained with Ansys Fluent solver we found that the constructed code operates correctly.

Keywords: oblique shock wave, moving curvilinear coordinate system, Van Lear method.

Introduction

Irregular shape nozzles as the parts of jet engines are wide spread in the various rocket and missile engineering fields nowadays. For instance, variable exit area nozzles optimizing the operation of jet engines under the number of various velocities and nozzle outflow regimes along with the deflectable thrust nozzles with the possibility of changing a thrust vector. During the rocket flight one can observe generation of diverse body and its components vibration: from mechanical vibration caused by rocket-engine starting to the vibration arising due to aerodynamic load while supersonic cruising (flatter).

1. Problem formulation

We consider the domain formed by the upper and lower solid boundaries, Figure 1. Gas inflows to the domain through the left boundary AB with supersonic velocity, which is inclined at an angle α, and drains away through the boundary CD.

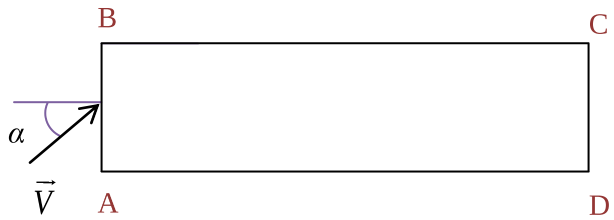

Fig.1. Computational domain

The upper wall BC make oscillations along a vertical axis according to the harmonic law $y = |DC| + A\frac{x}{|AD|}\sin\ \omega t$, wherein point B supposed to be fixed. On initial time we suggest the value of the domain inlet and outlet height is AB=DC=1, the domain length is AD=4. Gas flow is governed by the system of Euler equations in curvilinear coordinate system (1).

$$\frac{\partial\tilde{\mathbf{U}}}{\partial\tau}+\frac{\partial\tilde{\mathbf{F}}}{\partial\xi}+\frac{\partial\tilde{\mathbf{G}}}{\partial\eta}=0, \qquad (1)$$

where

$$\tilde{\mathbf{U}} = \begin{pmatrix} \rho J \\ \rho J u \\ \rho J v \\ \rho J E \end{pmatrix}, \quad \tilde{\mathbf{F}} = \begin{pmatrix} \rho(\alpha-\gamma) \\ \rho u(\alpha-\gamma) + p\cdot y_\eta \\ \rho v(\alpha-\gamma) - p\cdot x_\eta \\ \rho H(\alpha-\gamma) + p\cdot\gamma \end{pmatrix}, \quad \tilde{\mathbf{G}} = \begin{pmatrix} \rho(\beta-\sigma) \\ \rho u(\beta-\sigma) - p\cdot y_\xi \\ \rho v(\beta-\sigma) + p\cdot x_\xi \\ \rho H(\beta-\sigma) + p\cdot\sigma \end{pmatrix}, \tag{2}$$

u, v –components of velocity vector V, directed along x, y axes in Cartesian coordinate system; ρ – density; p – pressure; H – enthalpy, $E + p/\rho$; E – total energy, $\frac{1}{k-1}\cdot\frac{p}{\rho}+\frac{u^2+v^2}{2}$; k – heat capacity ratio, 1.4; $\alpha = u\cdot y_\eta - v\cdot x_\eta$; $\gamma = x_\tau\cdot y_\eta - y_\tau\cdot x_\eta$; $\beta = v\cdot x_\xi - u\cdot y_\xi$; $\sigma = y_\tau\cdot x_\xi - x_\tau\cdot y_\xi$.

Initial and boundary conditions at the domain inlet are set in the form (3).

$$\rho = 1;\ u = 2;\ v = 0.5;\ P = 1 \tag{3}$$

The boundary conditions at the exit are specified based on the value of the gas flow rate in the last cells. For supersonic flow, $u > c$ (where c is a speed of sound), the boundary conditions at the exit are not specified, otherwise, when the flow is not supersonic, at the right boundary we set the pressure. To obtain a solution to this problem the finite volume method is applied. The determination of flows through the faces of the computational cells was carried out using the Van Lear method. The original equation is a difference scheme with the first order of approximation in spatial and in time variables.

1. Implementation of Van Lear method.

To carry out the splitting procedure in a curvilinear coordinate system, the vector $\tilde{\mathbf{F}}$ (according to [3], [4]) will be represented as:

$$\tilde{\tilde{\mathbf{F}}} = \begin{pmatrix} \rho U_\eta \\ \rho U_\eta^{\,2} + p \\ \rho v_\eta U_\eta \\ \rho H U_\eta \end{pmatrix} + \begin{pmatrix} 0 \\ \rho U_\eta \cdot W_t \\ 0 \\ p\cdot W_t \end{pmatrix},$$

where

$$U_\eta = \frac{y_\eta(u - x_\tau) - x_\eta(v - y_\tau)}{\sqrt{x_\eta^{\,2} + y_\eta^{\,2}}};\quad u_\eta = \frac{u\cdot y_\eta}{\sqrt{x_\eta^{\,2} + y_\eta^{\,2}}} - \frac{v\cdot x_\eta}{\sqrt{x_\eta^{\,2} + y_\eta^{\,2}}};$$

$$v_\eta = \frac{u\cdot x_\eta}{\sqrt{x_\eta^{\,2} + y_\eta^{\,2}}} + \frac{v\cdot y_\eta}{\sqrt{x_\eta^{\,2} + y_\eta^{\,2}}};\quad W_t = \frac{\gamma}{\sqrt{x_\eta^{\,2} + y_\eta^{\,2}}}.$$

Vector $\tilde{\mathbf{G}}$ splitting should be carried out in a similar way.

2. The results of the numerical solution

The numerical simulation results of supersonic gas flow in a region with a time-varying geometry have been obtained on a difference mesh 200 × 50 and are presented using isobars and Mach number isolines. In the considered task, there are two characteristic times of the process - the oscillation period of the domain upper wall and the time of propagation of the sound wave along the region. Firstly, we give the consideration of the first case, when $t_{os} >> t_{gd}$.

Here $t_{gd} = L/c$ – gasdynamic characteristic time; L – characteristic dimension of the domain; $t_{os} = 1/\omega$ – characteristic oscillation period; ω – oscillation frequency.

In order to make clear the fact when this condition is fulfilled, the flow pattern mainly does not depend on the presence of oscillations of the domain upper wall, calculations have been made when the gas inflows at an angle $\alpha = 14^{\circ}$ for the flow in the region with a fixed upper wall (Fig.2) and the flow when the upper wall moves according to the law $y = 1 + 0.17\frac{x}{4}\sin\ 0.5t$, (Fig. 3, 4). Comparison of the position and intensity of oblique shock waves shows that a change in the position of the upper wall with time has virtually no effect on the overall flow pattern.

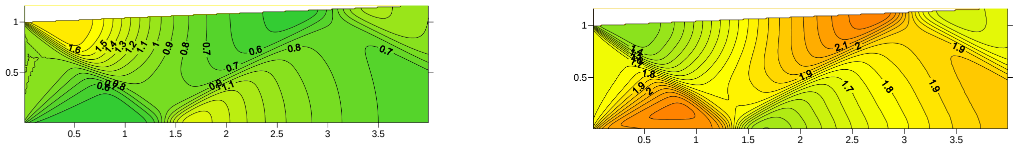

Fig.2. Pressure and Mach number distributions

a) $\frac{t}{T} = \pi$

b) $\frac{t}{T} = 2\pi$

c) $\frac{t}{T} = 3\pi$

d) $\frac{t}{T} = 4\pi$

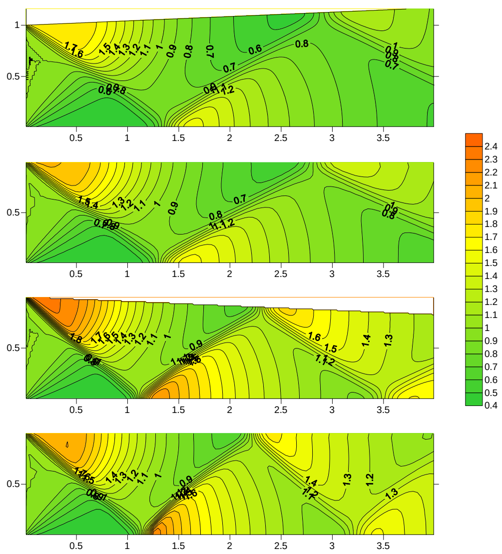

Fig.3. Pressure field

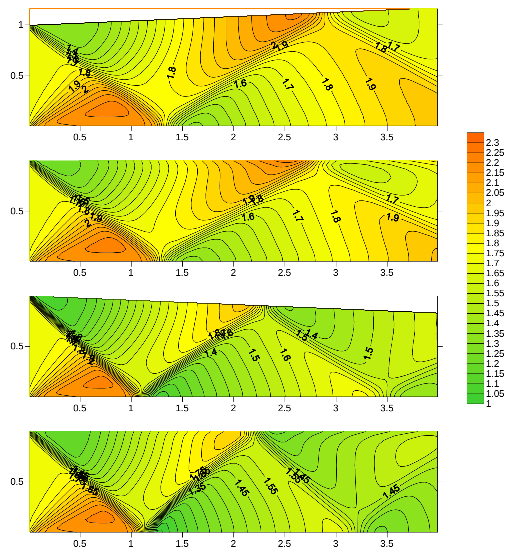

Fig.4. Mach-number distribution

Consider the following case $t_{os} \approx t_{gd}$. This case corresponds to the gas flow in the domain with the upper wall position, changing according to the law $y = 1 + 0.17\frac{x}{4}\sin\ t$. The calculation results of the pressure fields and the Mach numbers for gas flowing into the region at an angle $\alpha = 0°$ for the one period of time T are presented in Figures 5, 6, respectively. Figure 5 (a) clearly demonstrates that the rise of the upper wall causes the formation of rarefaction wave at the beginning of the region and, on the contrary, when the wall moves down - a compression wave, Figure 5 (c).

In order to establish the dependence of the intensity of shock waves on the law of wall displacement, we carried out a calculation in presence of direct gas inflow (i.e. $\alpha = 0°$) for a region with the upper wall moving according to the law $y = 1 + 0.51\frac{x}{4}\sin\ t$.

The results of this simulation are presented in Figures 7, 8. A comparison of Figures 5 and 7 allows us to conclude that with an increase in the swing amplitude of the upper wall, the intensity of generated rarefaction waves (Fig. 5 (a), 7 (a)) and compression waves (Fig. 5 (c), 7 (c)) also increases.

a) $\frac{t}{T} = \frac{\pi}{2}$

b) $\frac{t}{T} = \pi$,

c) $\frac{t}{T} = \frac{3\pi}{2}$

d) $\frac{t}{T} = 2\pi$

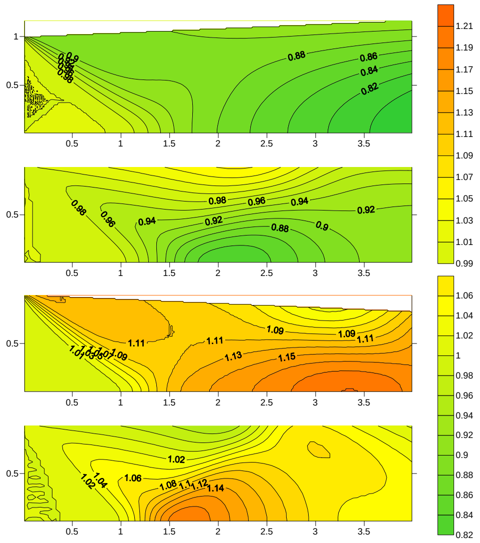

Fig.5. Pressure field

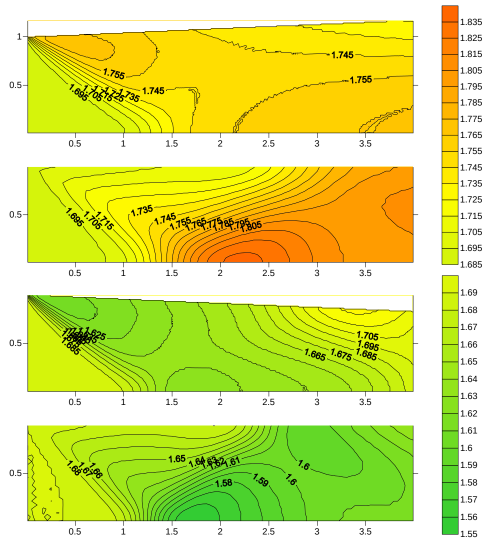

Fig.6. Mach-number distribution

a) $\frac{t}{T} = \frac{\pi}{2}$

b) $\frac{t}{T} = \pi$

c) $\frac{t}{T} = \frac{3\pi}{2}$

d) $\frac{t}{T} = 2\pi$

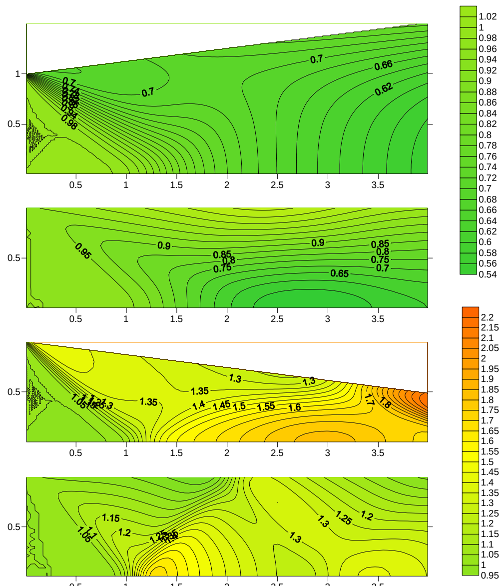

Fig.7. Pressure field

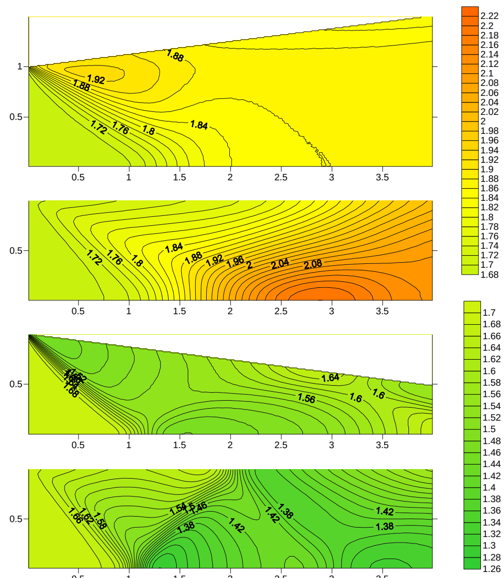

Fig.8. Mach-number distribution

Figures 9, 10 show the calculation results for the case when gas flows into the area at an angle $\alpha = 14°$. The movement of the upper wall is described by the law $y = 1 + 0.17\frac{x}{4}\sin t$. A comparison of Figure 9 with Figure 3 shows that with an increase in the upper wall oscillation frequency, the intensity of the resulting oblique shock waves weakens. In addition to the above, at the time $\pi/2$, when the upper wall reaches the maximum point (Fig. 9a), the shock waves do not have enough time to propagate along the entire length of the region.

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а) $\dfrac{t}{T} = \dfrac{\pi}{2}$

б) $\dfrac{t}{T} = \pi$

в) $\dfrac{t}{T} = \dfrac{3\pi}{2}$

г) $\dfrac{t}{T} = 2\pi$

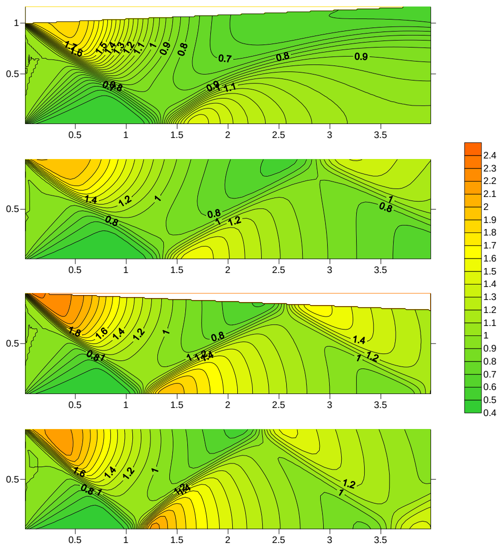


Fig.9. Pressure field

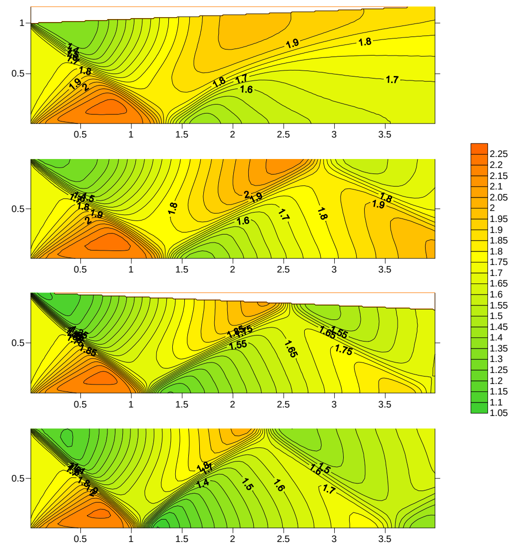
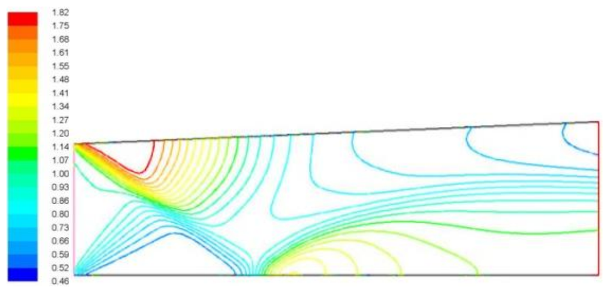


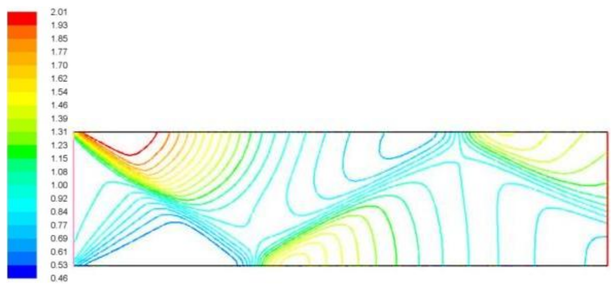
Fig.10. Mach-number distribution

3. The Ansys Fluent results

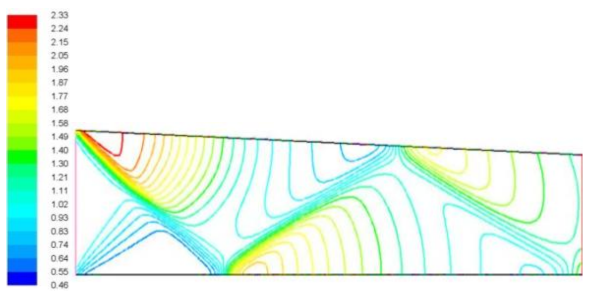
To verify the simulation results, we calculated the pressure fields and Mach numbers using the Ansys Fluent software. Here are the simulation results for gas flowing into the domain at an angle $\alpha = 14°$, (Fig. 11, 12).



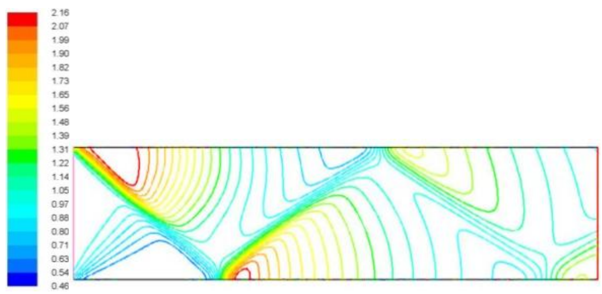
a)



b)



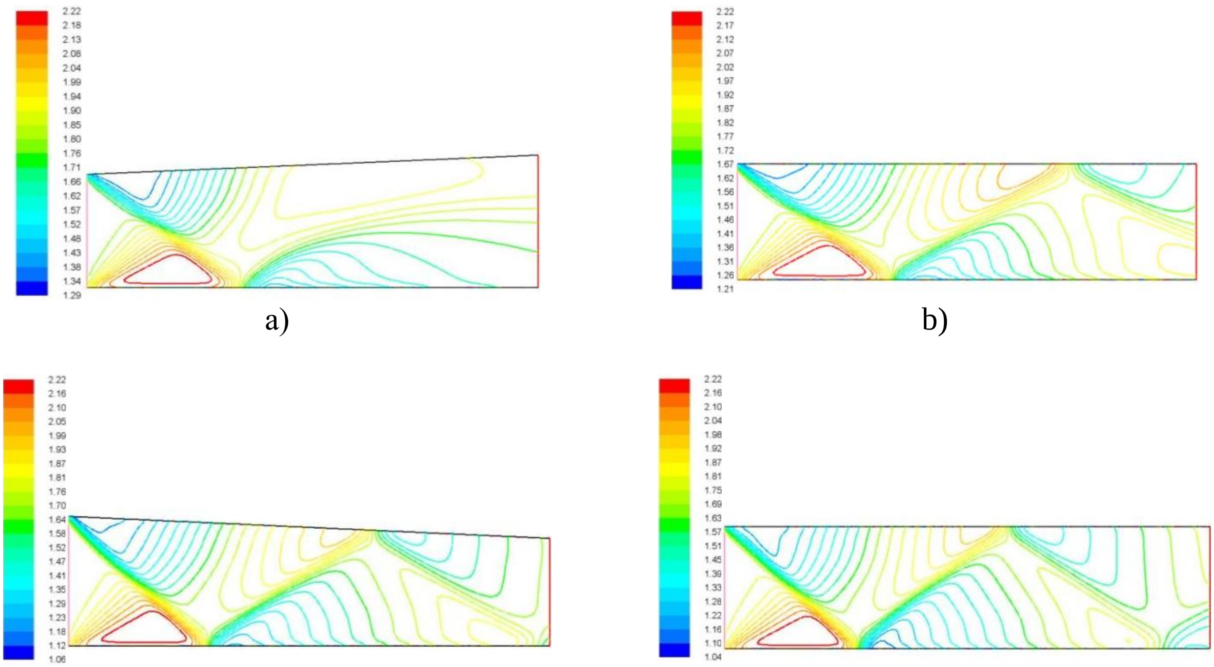
c)



d)

Fig.11. Pressure field as per Ansys Fluent

The law of motion of the upper wall is $y = 1 + 0.17\dfrac{x}{4}\sin\ t$. Comparison of the flow pattern, position and intensity of shock waves in Figures 9 and 10, 11 and 12 shows that the created procedure of calculating gas flow in a curvilinear region with moving boundaries allows us to obtain a physically correct numerical solution.



a) b) c) d)

Fig.12. Mach-number distribution as per Ansys Fluent

a) $\frac{t}{T}=\frac{\pi}{2}$, b) $\frac{t}{T}=\pi$, c) $\frac{t}{T}=\frac{3\pi}{2}$, d) $\frac{t}{T}=2\pi$

Conclusion

We have studied the features of the oblique shock wave propagation in a region in the presence of an oscillation of the upper boundary depending on the type of the harmonic law on which this oscillation is carried out. It was found that at certain amplitude, the oscillation of the wall practically does not affect the nature of the flow in the considered region. A comparison of the numerical solution obtained by the created program with the numerical solution obtained with the Ansys Fluent solver showed the correctness of the created program work.

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Article accepted for publication 19.11.2018