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CALCULATION OF THE END PLATES FRICTIONAL RESISTANCE EFFECT ON THE FLAT JET DYNAMICS

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This article is a continuation of scientific research on the calculation of the effect of friction resistance of end plates on the aerodynamics of a flat jet. In the previous works, the resistance was calculated for a turbulent boundary layer. This paper shows the results of calculating the effect of friction resistance of end plates on the patterns of development of a flat free jet. A flow diagram of the jet between the end surfaces has been constructed. The resistance calculation for the laminar boundary layer has been carried out. The formula for calculation of the change in the maximum jet velocity in a first approximation has been derived. The comparison of the theoretical calculations with the experimental data has shown a good agreement.

Keywords: three-dimensional jet, flat jet, end plates, frictional resistance, boundary layer

Introduction

Over the last few decades, the dynamic and oscillatory response of a free three-dimensional jet flowing from a rectangular outlet cross-section nozzle, on the main and partially on the initial section of the current, have been subjected to detailed research [1-5]. Recently, attention has also been paid to the development of the vortex structure and its effect on the development of turbulent and mean flow properties on the initial, transition and main sections of a free jet stream. In experimental installations in the study of a plane jet, as a rule, in order to exclude the effect of finiteness of the rectangular nozzle height, the flow field is limited by end plates installed in parallel to the direction of flow, as a continuation of the end walls of the rectangular nozzle outflow section. As we see here, due to the influence of the end walls, instead of a three-dimensional jet, we obtain a plane jet bounded by these side walls.

It can be said that the new experimental and theoretical data obtained provide broad information about the effect of end walls and large-scale coherent vortices on the development of turbulent jets that flow from a rectangular nozzle. For example, in work [6] the effect of friction resistance of the end plates on the patterns of a plane free jet was experimentally investigated. Recently, much attention has been given to the study of coherent structures of the wall jets flow [7]. This field is an important target of research. It is also important to continue research on dynamic flow properties. In this paper, as a continuation of experimental studies shown in [6], a theoretical calculation of end plates friction resistance effect on the development patterns of a free plane jet is performed.

1. Calculation of friction resistance face end plates

Calculation of the resistance effect on the end walls. To calculate the effect of resistance of the end walls on the flat jet attenuation, we will look at the following jet flow scheme between end surfaces. Fig. 1 shows the jet flow schemes bounded by the end flat walls, in the *xoy* plane. In the *xoy* plane the jet, as in the ordinary free jet, has an initial ("*i*" index), a transition ("*t*" index) and main sections and lateral free mixing boundaries, the nozzle width in the axis direction is $2b$. In the *xoz* plane, the $2h$ high jet flowing from the nozzle is bounded by the end plates from the sides in the direction of the *oz* axis. On the first section of the jet, after leaving the nozzle along the end walls, a laminar or turbulent boundary layer develops with a uniform profile along the *z* axis between the boundaries of the boundary layers. The development of these boundary layers is similar to the boundary layer when the plate is flowed past by a uniform flow. At the end of the 1-section, the boundary layers are joined on the jet axis and the 2-section of the jet begins, in which the flow in the *xoz* plane is analogous to the flow in a flat channel. Accordingly, the development of the boundary layer and the flow on the first section are analogous to the flow past the plate by a uniform flow, while in the second section it is analogues to a flow in a flat channel. The geometric parameter $\lambda=2h/2b$ characterizes the relative elongation of the outlet section of the nozzle.

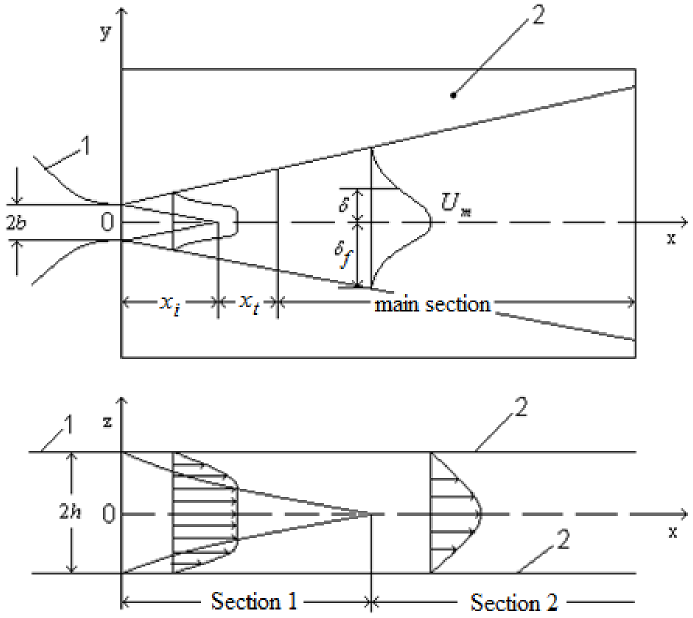

Fig.1. Diagram of a flat jet bounded by the end walls:
1 – nozzle; 2 – end plates. x, y, z – right angled Descartes coordinates; δ – current value of the boundary layer thickness; δ_c – free jet boundary layer thickness in y-direction. U_m – velocity on the jet axle

In view of the foregoing, on the first jet section we adopt the change in the boundary layer thickness along *z* on the end plates as the following dependences:

- for laminar flow

$$\delta_z = \frac{5.0 \times x}{\sqrt{\dfrac{U_m \times x}{\nu}}} \tag{1}$$

for a turbulent boundary layer

$$\delta_z = \frac{0.37x}{\left(\dfrac{U_m \times x}{\nu}\right)^{1/5}} \tag{2}$$

Here x is the longitudinal coordinate, U_m is the velocity at the jet axis, ν is kinematic viscosity, and $\frac{U_m \times x}{\nu}$ – we adopt as a Reynolds number $\mathrm{Re}_{mx} = \frac{U_m \times x}{\nu}$.

The length of the first section is determined from the condition of $x=x_1$ with $\delta_z = h$. Thus, in order to determine walls resistance, we can use the following formulas from the paper [6]:

$$\xi = \frac{16}{\text{Re}},\ \text{Re} = \frac{U_m \times 2h}{\nu} \tag{3}$$

$$C_f = \frac{0.664}{\sqrt{\text{Re}_{mx}}},\ \text{or}\quad C_f = \frac{0.0576}{\left(\frac{U_m \times x}{\nu}\right)^{0.2}}.$$

After the boundary layers come in contact for the second flow section, we apply the law of resistance in a flat channel with ξ hydraulic resistance coefficient for laminar flow:

$$\xi = \frac{16}{\text{Re}},\ \text{Re} = \frac{U_m \times 2h}{\nu} \tag{3a}$$

For turbulent flow:

$$\xi = \frac{0.3164}{\text{Re}^{\frac{1}{4}}},\ \text{Re} = \frac{U_m \times d_r}{\nu}, \tag{4}$$

where $d_r = \frac{4F}{\vartheta}$ is hydraulic diameter, defined as the ratio of the quadruple section area of channel F to its perimeter ϑ.

Below we provide an approximate calculation of the change in the total jet momentum under the effect of end walls resistance in the laminar jet flow.

2. Calculation of resistance at a laminar boundary layer

In the presence of end walls resistance, the total jet momentum is not conserved and decreases along the length of the jet [6]:

$$\frac{dK}{dx} = -2\int_{-\delta_f}^{\delta_f} \tau_W dy, \tag{5}$$

where K is the total flow momentum in any jet section, τ_w is the frictional stress on the wall at a y distance from the symmetry plane, and δ_f is the total half-width of the jet equal to the distance from axis to the outer boundary at $U = 0$.

Considering the jet between closely spaced plates with the Reynolds number being $\text{Re}_0 = \frac{U_0 b}{\nu} < 10^3$ (0 index is the value of the elongation parameter at the nozzle edge) and $\lambda < 1$; the first section length is small and the boundary layers on the walls contact at the axis already at a distance of $\frac{x_1}{b} < 10$. Therefore, for approximate calculation, we could adopt the profiles both along z axis and along y axis established from the end or from the nozzle exit; the plate resistance is calculated using the laminar boundary layer model.

For simplicity, we assume that along *oy* axis the velocity profile of the jet corresponds to the polynomial [6], and along *oz* axis – to the parabolic profile in the flat channel:

$$\frac{U}{U_1} = 1 - \frac{z^2}{h^2}, \tag{6}$$

where U_1 is the velocity value in the jet section with coordinates x, y at z=0, U is longitudinal velocity. The polynomial is represented as follows:

$$\frac{U_1}{U_m} = 1 - 6\eta^2 + 8\eta^3 - 3\eta^4, \tag{7}$$

.

where U_1 is the velocity at the boundary of the wall boundary layer at a distance of δ_z from the wall at the appropriate distances $\eta = \frac{y}{\delta_f}$ from *zox* plane in this section.

We use (7) in (6), then along the jet section the velocity distribution will look as follows:

$$\frac{U}{U_m} = \left(1 - \frac{z^2}{h^2}\right)\left(1 - 6\eta^2 + 8\eta^3 - 3\eta^4\right), \tag{8}$$

Total jet momentum in this section is as follows:

$$K = 4\int_0^h \int_0^{\delta_f} \rho U^2 dy dz = 1.579 h\delta\rho U_m^2 , \tag{9}$$

where ρ –is the liquid (gas) density.

Friction resistance of the walls on a section with dx length:

$$= \frac{16.576}{\mathrm{Re}_0\,\lambda} \frac{U_m}{U_0} \rho U_0^2 \delta dx \;\; 4dx \int_0^{\delta_f} \tau_w dy = 4dx \int_0^{\delta_f} \frac{24}{\frac{<U_1> 2h}{\nu}} \rho \frac{<U_1>^2}{2} dy = , \tag{10}$$

where τ_w is the frictional stress on the wall at a distance y from the plane of symmetry.

We use (9) and (10) in (5) and after generation we obtain the following:

$$\int_1^{\frac{U_m}{U_0}} \frac{d\left(\frac{U_m}{U_0}\right)}{\frac{U_m}{U_0}} = = -\frac{1}{2}\int_0^{\frac{x}{b}} \frac{d\left(\frac{x}{b}\right)}{\frac{x}{b}} - \frac{5.25}{\mathrm{Re}_0\,\lambda^2}\int_0^{\frac{x}{b}} \frac{d\left(\frac{x}{b}\right)}{\frac{U_m}{U_0}} , \tag{11}$$

where $\mathrm{Re}_0 = \frac{U_0 \times 2b}{\nu}$, here U_0 – is an initial flow velocity.

In the right part of (11) we use the null approximation solution

$$\int_1^{\frac{U_m}{U_0}} \frac{d\left(\frac{U_m}{U_0}\right)}{\frac{U_m}{U_0}} = ,$$

and after integration and exponentiation we obtain the following:

$$\frac{U_m}{U_0} = \frac{N}{\sqrt{\frac{x}{b} + \frac{x_0}{b}}} \exp\left[-\frac{10.50}{\mathrm{Re}_0\,\lambda^2 N}\sqrt{\frac{x}{b}}\right] = \frac{N}{\sqrt{\frac{x}{b} + \frac{x_0}{b}}} \exp\left[-a\sqrt{\frac{x}{b}}\right] , \tag{12}$$

where x_0 – is the pole distance, $a = \frac{10.50}{\mathrm{Re}_0\,\lambda^2 N}$.

In order to obtain a solution in the second approximation (12), we again use in the right part (11) and integrate. As a result, we obtain the following solution:

$$\ln\frac{U_m}{U_0} = -\frac{1}{2}\ln\frac{\frac{x}{b}}{\frac{x_i}{b}} - \frac{2}{a^2}\left[\exp\left[a\sqrt{x}\right] \times \left(\frac{1}{2}a^2 x - a\sqrt{x} + 1\right) - 1\right] , \tag{13}$$

where x_i – is the initial section length.

After (13) exponentiation we obtain:

$$\frac{U_m}{U_0} = \sqrt{\frac{\frac{x_i}{b}}{\frac{x}{b}}} \times \exp\left[-\frac{2}{a}\left[\exp\left[a\sqrt{x}\right] \times \left(\frac{a^2}{2}x - a\sqrt{x} + 1\right) - 1\right]\right] \tag{14}$$

,

In case of the values of $\frac{x}{b} \leq 100$ and $a\sqrt{x} << 1$ the function (14) can be represented as a following series:

$$\frac{U_m}{U_0} \approx \frac{N}{\sqrt{\frac{x}{b} + \frac{x_0}{b}}} \times \exp\left[-a\left(\frac{x\sqrt{x}}{3} + \frac{ax^2}{2}\right)\right] \tag{15}$$

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3. Discussion of results

Calculation comparison for (15) with the experimental data is shown in Fig. 2 for a jet at λ=0.62, U_0=9.2 and 44.1 m/s.

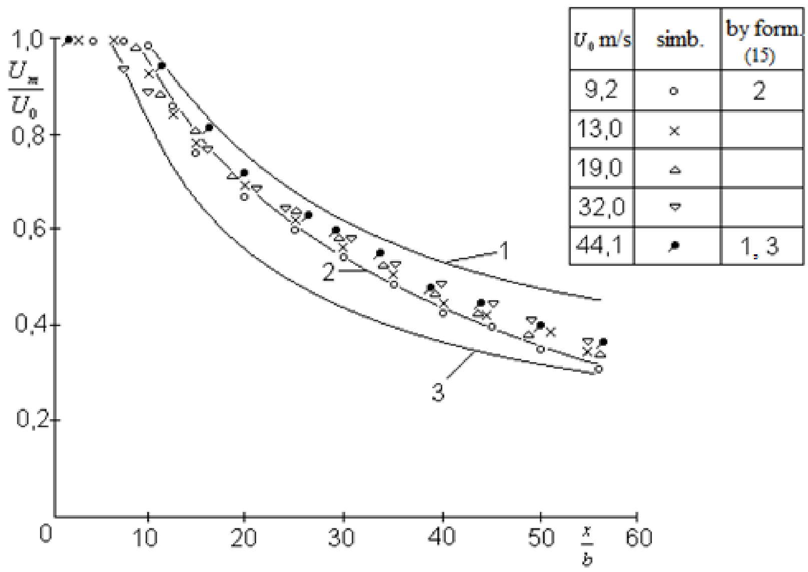

Fig.2. Dependence of $\frac{U_m}{U_0}$ on the distance from nozzle when λ=0.62.

Lines *1* and *2* calculation based on (15); line *3* calculation based on [6].

The same figure also provides calculations for the turbulent boundary layer from work [6] using the following formula:

$$\frac{U_m}{U_0} = \frac{N}{\sqrt{\frac{x}{b} + \frac{x_0}{b}}} \times \exp\left\{-\frac{0.1481}{A}\left(\frac{x}{b}\right)^{0.9} + \frac{0.01372}{A^2}\left(\frac{x}{b}\right)^{1.8} - \frac{0.00288}{A^3}\left(\frac{x}{b}\right)^{0.27}\right\} \tag{16}$$

,

where $A = \lambda \mathrm{Re}_0^{0.2} N^{0.2}$.

As can be seen, formula (15) much better corresponds to the experience data with λ=0.62, whereas the formula for the turbulent boundary layer is in better agreement with the experimental data with λ >1 [6].

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Conclusions

A jet flow diagram was constructed between the confining end walls. In the *xoy* plane, the jet propagates as a free jet. In the *xoz* plane in the first section, the boundary layer formed on the end walls is similar to the boundary layer when flowing past the plate as a uniform flow, and in the second section, when the boundary layers close on the axis of the flow, the flow is analogous to the flow in a flat channel. The paper shows the results of calculation of end plates friction resistance effect on the development patterns of a free flat jet. The resistance is calculated for a laminar boundary layer. A calculation formula is obtained that describes the change in the maximum jet velocity in the initial approximation. The results of calculation are compared with the experimental data.

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